Part III
Solution to Equation 1.9 Using Separability

Chapter 18

$r \approx r_H$ Application: Gyromagnetic Ratios

After separation of variables (section 17.6) the "r" component of equation 1.9 can be rewritten as:

$$\left[\left(\frac{dt}{ds}\sqrt{g_{oo}}\,m_p\right)+m_p\right]F-\hbar c\left(\sqrt{g_{rr}}\frac{d}{dr}+\frac{j+3/2}{r}\right)f=0 \qquad (18.1)$$

$$\left[\left(\frac{dt}{ds}\sqrt{g_{00}}\,m_p\right)-m_p\right]f+\hbar c\left(\sqrt{g_{rr}}\frac{d}{dr}-\frac{j-1/2}{r}\right)F=0 \qquad (18.2)$$

Using the above Dirac equation it is easiest to find the gyromagnetic ratios gy for the spin polarized F=0 case. Recall the usual calculation of rate of the change of spin S gives $dS/dt \propto m \propto gyJ$ from the Heisenberg equations of motion. We note that $1/\sqrt{g_{rr}}$ rescales dr in $\left(\sqrt{g_{rr}}\frac{d}{dr}+\frac{J+3/2}{r}\right)f$ in equation 18.1. Thus to have the same rescaling of r in the second term we must multiply the second term denominator (i.e.,r) and numerator (i.e., J+3/2) each by $1/\sqrt{g_{rr}}$ and set the numerator equal to 3/2+J(gy), where gy is now the gyromagnetic ratio. This makes our equation 18.1 compatible with the standard Dirac equation allowing us to substitute the gy into the standard $dS/dt \propto m \propto gyJ$ to find the correction to dS/dt.

Thus again:

$[1/\sqrt{g_{rr}}](3/2 + J) = 3/2 + Jgy$, Therefore for J= ½ we have:

$$[1/\sqrt{g_{rr}}](3/2+½) = 3/2+½gy = 3/2+½(1+\Delta gy) \qquad (18.3)$$

Then we solve equation 18.3 for gy and substitute it into the above dS/dt equation.

S States: Recall ε and Δε and S states from 4.12 and section 2.2. Noting in equation 4.12 the 1+ε cancels we get the gyromagnetic ratio of the electron with $g_{rr}=1/(1+\Delta\varepsilon/(1+\varepsilon))$ and $\varepsilon=0$ for electron. Thus solve equation 18.3 for $\sqrt{g_{rr}}=\sqrt{(1+\Delta\varepsilon/(1+\varepsilon))}=\sqrt{(1+\Delta\varepsilon/(1+0))}=\sqrt{(1+.0005799/1)}$. Thus from equation
18.3$[1/\sqrt{(1+.0005799)}](3/2 + ½) = 3/2 + ½(1+\Delta gy)$. Solving for Δgy gives gyromagnetic rato of the electron Δgy=.00116

Going to higher energies (so ε≠0 in equation 18.3) we get the gyromagnetic ratio of the muon.
$2P_{3/2}$ states: Recall the 2P3/2 states from chapter 3. Note also that k can be positive or negative since $4\pi k=Z_{00}$ in our Lagrangian with a positive k meaning at least one charge is not canceled.. Therefore $1/g_{rr} = 1 \pm k/r+\varepsilon$ (using our Frobenius solution expansion near $r \approx r_H$ of eq.19.5 below multiply through by $(1+\varepsilon/4)((1+\varepsilon+..) \approx 1+.08=1+\varepsilon'$ so a pion mass is then added to the protons) from the ±nature of Z_{oo}. Therefore we have two cases at the boundary r=k

CASE 1	$1/g_{rr} = 1+k/k+\varepsilon$	charge 1	(core case)
CASE 2	$1/g_{rr} = 1- k/k+\varepsilon$	charge 0	(use m from case 1)

Note: The effect of a zero charge is to make metric component g_{oo} ($=1/g_{rr}$) contribution zero in case 2. Thus the effect of *non*zero charge is to increase the dimensionality. This provides the reason that Kaluza Klein theory (adding a 5^{th} dimension) is so successful at injecting E&M into general relativity. But Kaluza Klein theory must be avoided at all costs because it adds unnecessary mathematical complexity. 2D is sufficient as we showed in Chapter 1.

CASE 1: Plus +k, therefore is the proton + charge component. $1/g_{rr} = 1+k/k + \varepsilon = 2+ \varepsilon$. Thus from equation 18.1, 18.2 $\sqrt{2+\varepsilon}$ (1.5+.5)=1.5+.5(gy), gy=2.8

The gyromagnetic ratio of the proton (therefore that above r≈ k stability was indeed proton stability as we concluded) mass=m_p . $dt/ds\sqrt{g_{oo}} = 1/\sqrt{g_{oo}} = E = m_p$

CASE 2: negative k, thus charge cancels, zero charge:

$1/g_{rr} = 1-k/k + \varepsilon = \varepsilon$ Therefore from equation 18.3 and case 1 $1/g_{rr} = 1+k/k+\varepsilon$:

$\sqrt{\varepsilon}$ (1.5+.5)=1.5+.5(gy), gy=-1.9,

the **gyromagnetic ratio of the neutron** with the other charged and neutral hyperon magnetic moments scaled using their masses by these values respectively.

Chapter 19

Frobenius series solution method applied to CASE1, CASE2

19.1 Series Solutions ψ Ansatz Near $r \approx r_H$

Recall equations 18.1, 18.2:

$$\left[\left(\frac{dt}{ds}\sqrt{g_{oo}}\,m_p\right)+m_p\right]F-\hbar c\left(\sqrt{g_{rr}}\,\frac{d}{dr}+\frac{j+3/2}{r}\right)f=0$$

$$\left[\left(\frac{dt}{ds}\sqrt{g_{00}}\,m_p\right)-m_p\right]f+\hbar c\left(\sqrt{g_{rr}}\,\frac{d}{dr}-\frac{j-1/2}{r}\right)F=0$$

Recall from the previous section $g_{oo}=1-k/r-(\varepsilon+\Delta\varepsilon)$. Also **recall our Dirac doublet** (equation 4.17 and section 1.5 originating in 1.2e) must have a left handed zero mass component will be called case 1 and case 3 respectively below. Also we need the equivalent of the singlet equation 1.2a is our below case 2. Also in equation 2.6 at $r=r_H$ the eigenvalue is $\Delta\varepsilon+\varepsilon+1=2m_p$ for that principle quantum which then must be the same for the $2P_{3/2}$ state Here we write out the left handed Dirac Doublet Eq.4.17 in the general representation of the Dirac matrices. Also recall from chapter 3 that the $2P_{3/2}$ state (and its sp^2 hybrid) for this new electron Dirac equation gives a azimuthal trifolium, 3 lobe shape and thus a λ/3 spherical harmonic wavelength so that for covalent bonding $r'\approx r_H/3$ in $\kappa_{oo}=1-r'/r$. This λ/3 also is used in section 16.1 to calculate P wave scattering.

To use the f & F components of the equation 18.1, 18.2 Dirac equation we write the Dirac equation for free particle motion along the symmetry axis z (r=ratio of momentum to energy) to find the chirality of the components in the general representation.of equation 4.17. We then compare this z motion free particle Dirac equation eigenfunction structure with radial component structure to arrive at a sense of which components of the radial equation are left handed and which aren't. This step is a little more complicated here because we are not using the chiral representation of the Dirac matrices, but the standard representation instead. In any case given that the electron is positive energy, then (as we see below) for the positron -E gives left handed f and F implying that this object *must* have a positive charge since this left handedness(doublet, 4.17) results from the fractalness (There is a corresponding argument for G and g). The proton indeed is positive charged. So:

$$\left[-\left(\frac{dt}{ds}\sqrt{g_{oo}}\,m_p\right)+m_p\right]g-\hbar c\left(\sqrt{g_{rr}}\,\frac{d}{dr}+\frac{j+3/2}{r}\right)G=0\rightarrow \mu c^2u_1+cpu_3-Epu_1$$

$$\left[-\left(\frac{dt}{ds}\sqrt{g_{00}}\,m_p\right)-m_p\right]G+\hbar c\left(\sqrt{g_{rr}}\,\frac{d}{dr}-\frac{j-1/2}{r}\right)g=0\rightarrow cpu_1-\mu c^2u_3-Epu_3$$

$$\left[-\left(\frac{dt}{ds}\sqrt{g_{oo}}\,m_p\right)+m_p\right]f-\hbar c\left(\sqrt{g_{rr}}\,\frac{d}{dr}-\frac{j-1/2}{r}\right)F=0\rightarrow \mu c^2u_2-cpu_4-Epu_2$$

$$\left[-\left(\frac{dt}{ds}\sqrt{g_{00}}\,m_p\right)-m_p\right]F+\hbar c\left(\sqrt{g_{rr}}\,\frac{d}{dr}+\frac{j+3/2}{r}\right)F=0\rightarrow -cpu_2-\mu c^2u_4-E_pu_4$$

where to get correspondence from these two Dirac equation structures we see that at +E: u^R= $\begin{pmatrix}1\\0\\r\\0\end{pmatrix}$ =g, u^L= $\begin{pmatrix}0\\1\\0\\-r\end{pmatrix}$ =f; -E: No (v^R= $\begin{pmatrix}r\\0\\1\\0\end{pmatrix}$ here), v^L= $\begin{pmatrix}0\\r\\0\\1\end{pmatrix}$ =F, Note in general (with r≈0) here: $\begin{pmatrix}ig\\if\\G\\F\end{pmatrix}=\begin{pmatrix}u^R\\u^L\\v^R\\v^L\end{pmatrix}$ =Ψ . So we have the solution that in the standard representation of the left handed doublet is given by F and f only for –E of the electron (here a positron needed below for + proton hadron excited states) at the horizon. Dirac matrices

So for the left handed doublet: $\begin{pmatrix}F\\f\end{pmatrix}_L$ we have respectively $\begin{matrix}q\pm,\ \ m\neq 0,-E,\ forF\\ q=0, m=0,+E,\ forf\end{matrix}$ (19.4)

Or more succinctly equation 1.9 in the Dirac doublet form implies:

CASE 1	$1/g_{rr}$ =1+k/k+ε	F	charge 1, m=1 (core case)
CASE 2	$1/g_{rr}$ =1- k/k+ε	F	charge 0, m from case 1)
CASE 3		f	charge 0, m=0

We solve these equations only near $r\approx k_H$ since that is where the stability is to be found (and also fortunately were these equations are *linear* differential equations). Thus our first step is to expand $\sqrt{g_{rr}}$ about this radius and drop the higher order terms.

The Frobenius series solution method can now be used to solve equations 18.1 and 18.2 at $r\approx r_H$. See for example Mathematical Methods of Physics, Arfken 3rd ed. Page 454. First we solve the f in equation 18.1, plug that into equation 18.2 and then have an equation in only F. There we substitute a series solution ansatz $F=\sum a_n r^n$ in the resulting combined equations. We can then separate out the results into coefficients of respective rn and get recursion relations that will give us series that must be terminated at some N. Note the energy Eigenvalue 'E' will be in this series as $dt/ds\sqrt{g_{oo}}$ so we can then solve for the mass energy of these hadrons at specific J. We will need an indicial equation for the first term to start out this process. Also in this Frobenius solution method 'n' turns out to be a multiple of ½ and the series must start at n=-1. Finally to get the charge zero case the charged case must be done first and its constant masses used in the uncharged state calculations.

The method appears to be working correctly. For example at the ∑ particle mass there is a square root that gives two charges instead of the usual 1. Also the L=1 solution has a mass of two protons associated with it, giving that (otherwise very mysterious) deuteron L=1 eigenstate.

In any case **<u>all we are doing here is solving equation 1.9</u>** for the left handed Dirac doublet case (our section 1.5 and equation 4.11 fundamental use of the Dirac equation 1.9) inside the horizon r_H and just working out the math details of what has to be done to accomplish this. The energy in $\sqrt{1+2\varepsilon+\Delta\varepsilon}$ is split between two 2P lobes, as with the deuteron, each lobe giving a proton mass.[m= (tauon+muon)/2]. Thus we divide by two here: the proton mass is taken to be the unit mass. Not that r=k is inversely proportional to m_p .(i.e., $m_p \alpha 1/k_H$) For m_p=.5 then k_H=2 for *each* of these 2P lobes. In the energy square roots (below) then we can normalized out the .5 (because k_H squared is always reciprocal of m_p^2 in the energies) so the proton mass energy becomes unit 1 in all the rest of these calculations.

19.2 CASE 1 Excited States for F, m≠0, q±

Again case 1 is one of the equation 18.1 possibilities. Therefore let R=k_H-r, r<<R (for stability) we can write in 18.1:

$$\sqrt{g_{rr}} = 1/\sqrt{1+k_H/R+\varepsilon} \approx \frac{\sqrt{R}}{\sqrt{R+k_H+R\varepsilon}} = \qquad (19.1)$$

$$\frac{\sqrt{k_H - r}}{\sqrt{k_H - r + k_H + (k_H - r)\varepsilon}} = \qquad (19.2)$$

$$\frac{\sqrt{k_H - r}}{\sqrt{k_H(2+\varepsilon) - r(1+\varepsilon)}} = \qquad (19.3)$$

$$\frac{(1-\varepsilon/4)}{\sqrt{2}}\left(\frac{1-\frac{r}{2k_H}+\frac{r^2}{8k_H^2}+..}{1-\frac{r}{4k_H}+\frac{r^2}{16k_H^2}+..}\right) = \qquad (19.4)$$

$$((1-\varepsilon/4)/\sqrt{2})\ (1-\frac{r}{4k_H}+\frac{3r^2}{32k_H^2}-..) \approx \frac{1-\frac{r}{4k_H}}{\sqrt{2}} \qquad (19.5)$$

Note that including the above 1±ε/4 the compensating (1±ε/4) in the next r term has the effect of a multiplying the derivative terms by 1±ε/4. This rescales r to allow us to still say that the stable boundary is still at r_H. Thus we could use it to also rescale t in the first term of equations 18.1 and 18.2 or note that (1+ε/4) (1+ε)=1+5/4ε thus renormalizing 1+ ε to 1+4/3ε =1+ε' everywhere. Also the $3r^2/32k_H^2$ terms must be included. We drop these perturbative terms until the end. Therefore substituting in equation 19.5 we find that equation 18.1 reads:

$$[E+m_p]F - \hbar c\left((1-r/k_H 4)\frac{d}{\sqrt{2}dr} + \frac{j+\frac{3}{2}}{k_H - r}\right) f = 0 = \qquad (19.6)$$

$$[E+m_p]F - \hbar c\left((1-r/k_H 4)\frac{d}{\sqrt{2}dr} + (1+r/k_H)(j+\frac{3}{2})/k_H\right) f = 0\text{; also:}$$

$$[E-m_p]f + \hbar c\left(\sqrt{g_{rr}}\frac{d}{dr} - \frac{j-\frac{1}{2}}{r}\right)F = 0$$

$$[E - m_p]f + \hbar c\left((1 - r/k_H 4)\frac{d}{\sqrt{2}dr} - (1 + r/k_H)\left(j - \frac{1}{2}\right)/k_H\right)F = 0$$

Therefore

$$f = -\hbar c\frac{\hbar c}{[E - m_p]}\left((1 - r/k_H 4)\frac{d}{\sqrt{2}dr} - (1 + r/k_H)\left(j - \frac{1}{2}\right)/k_H\right)F$$ substituting into

$$[E + m_p]F - \hbar c\left((1 - r/k_H 4)\frac{d}{\sqrt{2}dr} + \frac{j + \frac{3}{2}}{k_H - r}\right)f = 0 \qquad (19.7)$$

We find solving for f and substituting back in:

$$[E + m_p]F - \hbar c\left((1 - r/k_H 4)\frac{d}{\sqrt{2}dr} + (1 + r/k_H)(j + 1.5)/k_H\right)\bullet$$

$$\frac{\hbar c}{E - m_p}\left(-(1 - r/k_H 4)\frac{d}{\sqrt{2}dr} + (1 + r/k_H)(j - \frac{1}{2})/k_H\right)F = [E + m_p]F +$$

$$\frac{(\hbar c)^2}{(E - m_p)\sqrt{2}}\left(-(1 - r/4k_H)\frac{d}{k_H 4\sqrt{2}dr} + (1 - r/4k_H)^2\frac{d^2}{\sqrt{2}dr^2} - (1 - r/4k_H)(j - \frac{1}{2})/k_H^2\right)F$$

$$+\frac{(\hbar c)^2}{E - m_p}\left((1 + 3r/k_H 4)(j + 1.5)\frac{d}{\sqrt{2}k_H dr} - (1 + r/k_H)^2(j + 1.5)(j - \frac{1}{2})/k_H^2\right)F =$$

$$\left([E + m_p] + \left[\frac{(\hbar c)^2}{(E - m_p)}\left(-\left(j + \frac{3}{2}\right)(j - \frac{1}{2})/k_H^2 - (j - \frac{1}{2})/\sqrt{2}k_H^2\right)\right]\right)F +$$

$$\frac{(\hbar c)^2}{(E - m_p)\sqrt{2}}\left(-2\sqrt{2}\left(j + \frac{3}{2}\right)(j - \frac{1}{2})/k_H^3 + (j - \frac{1}{2})/4k_H^3\right)rF +$$

$$\frac{(\hbar c)^2}{(E - m_p)\sqrt{2}}\left(\frac{-1}{k_H 4\sqrt{2}} + \left(j + \frac{3}{2}\right)\frac{1}{k_H}\right)\frac{dF}{dr} +$$

$$\frac{(\hbar c)^2}{(E - m_p)\sqrt{2}}\left(\frac{1}{k_H^2 16\sqrt{2}} + \left(j + \frac{3}{2}\right)\frac{3}{4k_H^2}\right)r\frac{dF}{dr} +$$

$$\frac{(\hbar c)^2}{(E - m_p)\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)\frac{d^2F}{dr^2} +$$

$$\frac{(\hbar c)^2}{(E - m_p)\sqrt{2}}\left(\frac{-1}{2\sqrt{2}k_H}\right)r\frac{d^2F}{dr^2}$$

Here $r=2k_H$ is a regular singular point. Next substitute in $F = \sum_n a_n r^n$ with again half integer n allowed as well:

$$\sum_M^N\left(\left[E+m_p\right]+\left[\frac{(\hbar c)^2}{(E-m_p)}\left(-\left(j+\frac{3}{2}\right)(j-\frac{1}{2})/k_H^2-(j-\frac{1}{2})/\sqrt{2}k_H^2\right)\right]\right)a_n r^n + \qquad (19.8)$$

$$\sum_M^N\frac{(\hbar c)^2}{(E-m_p)\sqrt{2}}\left(-2\sqrt{2}\left(j+\frac{3}{2}\right)(j-\frac{1}{2})/k_H^3+(j-\frac{1}{2})/4k_H^3\right)a_{n-1}r^n + \qquad (19.9)$$

$$\sum_M^N\frac{(\hbar c)^2}{(E-m_p)\sqrt{2}}\left(\frac{-1}{k_H 4\sqrt{2}}+\left(j+\frac{3}{2}\right)\frac{1}{k_H}\right)(n+1)a_{n+1}r^n + \qquad (19.10)$$

$$\sum_M^N\frac{(\hbar c)^2}{(E-m_p)\sqrt{2}}\left(\frac{1}{k_H^2 16\sqrt{2}}+\left(j+\frac{3}{2}\right)\frac{3}{4k_H^2}\right)na_n r^n + \qquad (19.11)$$

$$\sum_M^N\frac{(\hbar c)^2}{(E-m_p)\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)(n+2)(n+1)a_{n+2}r^n + \qquad (19.12)$$

$$\sum_M^N\frac{(\hbar c)^2}{(E-m_p)\sqrt{2}}\left(\frac{-1}{2\sqrt{2}k_H}\right)(n+1)na_{n+1}r^n = 0 \qquad (19.13)$$

Note from equation 19.12 that this series diverges. To terminate the series we now take 19.8 and 19.11 together and 19.10 and 19.13 together (since they have the same a_n). For example combining the equation 19.8 and 19.11 terms

$$\left(\left[E+m_p\right]+\left[\frac{(\hbar c)^2}{(E-m_p)}\left(-\left(j+\frac{3}{2}\right)(j-\frac{1}{2})/k_H^2-(j-\frac{1}{2})/\sqrt{2}k_H^2\right)\right]\right)+$$

$$\frac{(\hbar c)^2}{(E-m_p)\sqrt{2}}\left(\frac{1}{k_H^2 16\sqrt{2}}+\left(j+\frac{3}{2}\right)\frac{3}{4k_H^2}\right)n,$$

Replacing the normalization $m_p \rightarrow m_p\ (1\pm\varepsilon\ )$ (from section 2.1):

$$\left(E^2-m_p^2\right)+\left(\left[\left(-\left(j+\frac{3}{2}\right)(j-\frac{1}{2})/k_H^2-(j-\frac{1}{2})/\sqrt{2}k_H^2\right)\right]\right)-\frac{1}{\sqrt{2}}\left(\frac{1}{k_H^2 16\sqrt{2}}+\left(j+\frac{3}{2}\right)\frac{3}{4k_H^2}\right)N=0$$

Therefore after rearranging:

$$E=\sqrt{m_p^2+\frac{1}{k_H^2}\left(j^2+1.7071j-1.10355-(j.5303+.8269)N\right)}\ , \qquad (19.14)$$

We have for a general Laurent series ansatz*:*

$$..+a_{-1}r^{-1}+a_{-1/2}r^{-1/2}+a_o r^o+a_{1/2}r^{1/2}+a_1 r^1+..=F$$

Note also that equations 19.8-19.13 imply that the coefficients of a given r^n are independent. Thus adding together the coefficients of r^n for equations 19.8–19.13 at a given n:

19.9(j-½)a_{n-1}+(19.8+19.11)a_n+(19.10+19.13)(n+1)a_{n+1}+19.12(n+2)(n+1)a_{n+2}=0 (19.15)

Method of Solving Equation 19.15
For the outside observer an F=0 finite boundary condition at infinity applies for flat vacuum value n=0, j=½ and for r^o, $r^{-½}$, r^{-1} and for complete vacuum for N=0, J=0.
Here then the generalized Laurent series $..+a_{-1}r^{-1}+a_{-1/2}r^{-1/2}+a_o r^o+a_{1/2}r^{1/2}+a_1 r^1+..=F$
reduces to $..+a_{-1}r^{-1}+a_{-1/2}r^{-1/2}+a_o r^o=F$. Thus either set 19.9(j-½)a_{n-1} =0 or (19.10+19.13)(n+1)a_{n-1}+19.12(n+1)(n+1)a_{n+2} =0 separately in eq.19.15 or set both equal to zero:

J= ½, sets eq.19.9=0

1) N=-1, in equation 19.14 gives mass eigenvalue for Ξ
Exact solution for all possible a_n, sets none of them to zero.
2) N=0, in equation 19.14 gives mass eigenvalue for *nucleon.* $dr^o/dr=0$ so all derivative of F terms are then zero and this solution applies inside as well.
N=0 flat J=0 allowed flat vacuum gives $\pi^{\pm}$ and with free e, j= ½ muon.
3) N= -½, in equation 19.14 gives mass eigenvalue of two Σ s since a plus *and* minus square root of r.

These 19.9=0 cases have case 2 zero charge representations as well.

N=-1, Principle QM number Also a_{-2} =0

1) J=0, in equation 19.14 gives mass eigenvalue for *K*
2) J=1, gives deuteron mass eigenvalue (bonding) given N=0,J=0 fills first (i.e., pion). Thereafter use nuclear shell model-Schrodinger equation many body techniques with these nonrelativistic lobes with this (bound state) force acting like a outer layer surface tension, finite height square well potential . Get a aufbau principle that then gives the D,F,G,..nuclear shell model states. Alternatively can fill that first S state in with free $1S_{½}$ (next state to filled state) and we have j= 3/2 Ω^- filling in some (i.e., uds) of the $2P_{3/2}$ states (see Ch.3) and thereby also deriving from first principles Gell Man's 1963 eight fold way for hyperon eigenvalue classification (to finish that effort need case II zero charge and case III Λ_o as well). M_p is replaced by 2 in c hyperons, by 4 for b hyperons as indicated in fig. 16-1 for how to fill in the cbt 2P harmonic states given the requirement to use r^2 then.

Also, to include higher order r expansion term effects in equation 19.5 we must include those perturbative 1+ε/4 and $3r^2/32k_H^2$ contributions which gives a n(n-1)/6.4 added to the "n" term component inside the radical of equation 19.14.
In our new pde δJ=o through LS spin-orbit coupling so the three spin½s and the L=1 add to a minimum. 1-½-½+½ =½ =S for the proton with possible Pauli principle non S=½ possibilities for larger mass eigenvalue.

Details of Above Solutions for Case 1

Thus besides the ground state (N=0 $F_{groundstate}=\sum a_n r^n = a_0 r^{-0}=F_0$ proton) we have the two solutions:
$F_{N=-1}=\sum a_n r^n = a_{-1}r^{-1}=F_1$., j= ½, 0., $F_{N=-½}=\sum a_n r^n = a_{-½}\, r^{-½}=F_2$. For j = ½. 0.

Note the energy eigenvalues (E) can be found from the solution to equation 19.14 and k_H =1 with E=1=938MeV. Thus

N= **0**, j= ½ then 19.14 gives +**Nucleon** (ground state) mass eigenvalue. Note that for the N=0, (with J= ½ and also J= 0 in section 19.5) ground state that the charge density is uniform (i.e., $\rho=K\propto r^0$) for r<k.

N=-½ , j=½ **two** valued because of the *two* square root solutions. Equation 19.14 then gives ∑± (charged sigma particle) 1184Mev particles, F_2 eigenfunction(s). Actual 1189Mev

N=-**1**, j= ½ gives *one* charged Ξ particle. Therefore the energy from equation 19.14 is 1327 Mev (actual 1321), F_1 eigenfunction.

Case 2 and case 3 give the neutral hyperons and Λ_o respectively (see case 3 below).

19.5 Nucleon Wavefunction: J=1, q≠0, N=-1 of Case 1

Here we recall case 1, section 19.3 above and compute energy eigenvalues for J=0 and J=1.again using equation 19.14 in case 1.

J=0

N=-1, j=0 E=490 MeV from equation 19.14 case 1. $K^{\pm}$. Substitute into strangeness equation 19.34 case 1 we obtain strangeness =1.

N=0, j=0 then from equation 19.14 *E=139.7 MeV* case (note again m=1+ε=1.061 in 19.14 for outside). This is the nontrivial F zero point energy (and so has a fundamental harmonic) for r<k since the square root in equation 20.1 becomes imaginary then. Thus the mass of $\pi^{\pm}$ is now the vacuum (e.g., note $F\propto r^o$ for N=0 here) ε'at r≈k explaining why this fundamental harmonic result for π is used in all the successful nuclear force theories such as in the Skyrmion Lagrangian for example. Note that:

$$m_{\pi\pm}=139\text{Mev}=1.3(105.6\text{MeV})=1.3\varepsilon=.08=\varepsilon' \qquad (19.22)$$

N=-1,J=1 case 1. Recall for J=1 we have $\psi \propto r\sin\theta \propto Y^1_1(\theta,\phi)$ double lobe ψ*ψ along the z axis: From equation 19.14 we find with these inputs that E=1867Mev implying that (because $E\sim 2m_p$ *and* J=1) this eigenstate is responsible for the spin 1 deuteron (state). The L=1, 2P state solution(s) are symmetric and so of the form $(1/\sqrt{2})(\psi_1\psi_2+\psi_1\psi_2)=\psi_s$ and have positive parity even if the 2P ψ_1 *and* ψ_2 each has negative parity. The Deuteron thus has + parity (Enge, 1966). Recall if we include the background metric in eq.4.11 $\kappa_{oo}=1+r_H/r+2\varepsilon'+\Delta\varepsilon$ and $\kappa_{rr}=1/(1+r_H/r+\Delta\varepsilon)$. So rescaling r→r-ε' =r'for r near r_H allows us to use our above solutions again. So in equation 18.1 $1/\sqrt{\kappa_{rr}}=\psi=1/\sqrt{(1+r_H/r'+\Delta\varepsilon)}\psi\approx 1/\sqrt{(1+r_H/r)}\psi+(\varepsilon/2)\psi$. Note if we again rescale our numerator J=1→1+(ε'/2)2 so that we have perturbed our Y_1 spherical harmonic with a $(\varepsilon/2)Y_2$ giving a measure of the oblate, non spherical structure (e.g quadrupolar ψ_D and higher. ε'/2 ≈.04 from 19.22 therefore the nonspherical component of ψ is approximately 4% of the total ψ and is often called the tensor component of the Deuteron eigenstate (Enge, 1966). This simplest multiparticle state represents the *deuteron* state and this is then the explanation for the deuteron tensor component of the nuclear force.

Also the energy of the Deuteron is given just outside the r_H boundary (so ε'→iε in 4.11) by $E_D=\text{Rel } 1876/\sqrt{\kappa_{oo}}=\text{Rel } 1876/\sqrt{(1+i\varepsilon')}+..=1876(1-i\varepsilon'/2+(3/8)(i\varepsilon')^2+..)$. So the added real term due to the ε' is equal to $1876(3/8)\varepsilon'^2=1876(3/8)(.08)^2=4\text{MeV}$. In free space ε'=0 and just outside the nucleus it gives this contribution to the Deuteron energy. Thus this $(3/8)\varepsilon'^2$ is the binding energy of the Deuteron.

Note from the equation 19.15 discussion for N *not* -1 we can only use J=1/2 and J=3/2 thus are restricted for two particles to S and P states (i.e. ½ + ½ =1) which then gives us the hyperons. For N=-1 we can use other J and can thereby construct large nuclei.
The multinucleon nuclei really are the solutions of the indicial equations of 19.15.
Recall in the shell model a hard shell nuclear outer wall is assumed with free space oscillations allowed inside this shell. The solutions to the Schrodinger equation are then spherical Bessel functions with corrections for spin orbit interaction, finite well height and tapered wells (Herald Enge, Introduction To Nuclear Physics, P.145). In any case an infinite mean free path for these oscillations is assumed to exist inside this shell. So how can there be an infinite mean free path inside this extremely high mass density region?
In that regard the above 2, J=1, N=-1 2P deuteron state can also be viewed as yet another Bogoliubov pairing interaction (such as in the SC section 17.1) giving this infinite mean free path of the electron pairs comprising a pion acting as a Cooper pair, just as in SC In the context of the section 17.1 pairing interaction model $A(dv/dt)/v^2$ is no longer as small but dv/dt becomes very large to due to the ultrarelativistic motion of the electrons inside the nucleons. In any case this infinite mean free path for these oscillations (recall Cooper pairs have an infinite mean free path) is thereby explained here as a new type of superconductivity.

Spin Orbit Interaction In Shell Model

If equal numbers of Neutrons and Protons gyromagnetic ratios then gy_P-gy_N =2.7-1.9 =.8.
Since more neutrons in heavier elements: (1/1.1)(.8)=.7.
$R=r_H \equiv ½$ Fermi measured from singularity at 1-½ = ½ .
From 2P3/2 at $r=r_H$ Fitzgerald contraction discussion in section 2.2: $r \rightarrow R = ½(1-½) = ¼$ Fermi $\equiv R_V(r-r_H)$ so $R_v(r-r_H) \rightarrow Kr$. From chapter 14 $V=1/(r-r_H)$. Spin orbit interaction=
$a_o^2(1/r)(\partial V/\partial r)(s \bullet L)$=

$$a_0^2 \frac{1}{R_V(r-r_H)} \frac{\partial V}{\partial (R_V(r-r_H))}(s \bullet L) = \frac{.7}{R_V(r-r_H)}\left(\frac{-1}{(R_V(r-r_H))^2}\right)(s \bullet L) = .7(4^3)(s \bullet L)\frac{1}{r}\frac{\partial V}{\partial r} =$$

$$= .7(64)(s \bullet L)\frac{1}{r}\frac{\partial V}{\partial r} = a_o^2 \frac{1}{r}\frac{\partial V}{\partial r}(s \bullet L) =$$ 45*E&M spin orbit interaction.

Thus the a_o=1Fermi. Thus the nuclear spin-orbit interaction is much larger than the E&M spin orbit interaction because the nucleons are much closer to r_H than to r=0 and the Fitzgerald contraction of the nucleon $2P_{3/2}$ state is on the order of ½.

At close range there are higher energies available so the 4mev (=be) in equation 19.3 (if we include r^2 contributions) becomes the binding energy for the deuteron in g_{oo}=1-k/r+be in 18.1 particles, F_2 eigenfunction(s). Actual 1189Mev
N=-**1**, j= ½ gives *one* charged Ξ particle. Therefore the energy from equation 19.14 is 1327 Mev (actual 1321), F_1 eigenfunction.
Case 2 and case 3 give the neutral hyperons and Λ_o respectively (see main Frobenius series solution paper).
The multinucleon nuclei are the solutions of the indicial equations of 19.15.
Recall in the shell model a hard shell nuclear outer wall is assumed with free space oscillations allowed inside this shell. The solutions to the Schrodinger equation are then spherical Bessel functions with corrections for spin orbit interaction, finite well height and tapered wells (Herald Enge, Introduction To Nuclear Physics, P.145). In any case an infinite mean free path for these

oscillations is assumed to exist inside this shell. So how can there be an infinite mean free path inside this extremely high mass density region?
In that regard the above 2, J=1, N=-1 2P deuteron state can also be viewed as yet another Bogoliubov pairing interaction (such as in the SC section 17.1) giving this infinite mean free path of the electron pairs comprising a pion acting as a Cooper pair, just as in SC In the context of the section 17.1 pairing interaction model $A(dv/dt)/v^2$ is no longer as small but dv/dt becomes very large to due to the ultrarelativistic motion of the electrons inside the nucleons. In any case this infinite mean free path for these oscillations (recall Cooper pairs have an infinite mean free path) is thereby explained here as a new type of superconductivity.

Particle Lifetimes

Recall from section 1.1: $\kappa_{oo}=1-r_H/r$ so $r-r\kappa_{oo}=r_H$ analogous to $dr-ct\kappa_{oo}=ds$ so $r_H=ds\equiv|dZ|$. From section 1.5 there are three Dirac equation contributions with one being the ultrarelativistic m_ν contribution. For that contribution we put Dirac αs into dr+idt=dZ the free space Dirac equation. Dividing by ds gives mass on the right side in that Dirac equation. Because the motion of the m_ν =1eV (section 16.5) particle is ultrarelativistic in these hadrons we apply figure 1-1 dr=dt so $\theta=45°$ and so $dZ/ds=e^{i\pi/4}dr/ds$ for the ultrarelativistic m_ν (on earth contribution of section 16.5). Note that $(e^{i\pi/4})^2=i$. We add another contribution (for spin ½, N=-1) to get zero charge case II below. For added $2P_{1/2}$ (K,±π mesons) there are 3e in r_H below (sect.20.3). Thus we obtain:

hyperons, **Kaons** and **±π**: $e^{i\pi/4}2e^2/m_\nu c^2= e^{i\pi/4}r'_H=R_H$

Recall that domain $r=r_H$ was the most stable, the proton state. This stability condition can be restated in terms of excess energy above the proton rest mass. Next substitute this m and ultrarelativistic m_ν in the r_H in equation 19.14 with this r'_H in the relativistic solution of equation 1.9 described in section 1.5.

$$E=\sqrt{m_p^2+\frac{1}{R_H^2}\left(j^2+1.7071j-1.10355-(j.5303+.8269)N\right)}$$

$$\approx m_p\left(1+\frac{\left(e^{i\pi/4}\right)^2\left(j^2+1.7071j-1.10355-(j.5303+.8269)N\right)}{2m_p^2r'^2_H}\right).$$

Add to above to 19.14 result to get for the total energy:

$$m_p\left(1+\left(\left(\frac{e^{i\pi/4}}{r'_H}\right)^2+\left(\frac{1}{r_H}\right)^2\right)\frac{\left(j^2+1.7071j-1.10355-(j.5303+.8269)N\right)}{2m_p^2}\right)$$

Plug $(\hbar c/e^2)^2=(1/\alpha)^2$ back in eq.18.1 and normalize $m_\nu c^2$ to 1/hz with 1/h. Next plug into the time propagator e^{iHt} and get for the r'_H (decay) term:

$$=\exp i\left(\left(\left(m_pc^2/h\right)+\left(e^{i\pi/4}\right)^2\left(m_\nu c^2/h\right)\frac{m_\nu}{m_p}\left(j^2+1.7071j-1.10355-(j.5303+.8269)N\right)\right)\left(\frac{\hbar c}{2e^2}\right)^2\right)t$$

$$=\exp i\left(\left(\left(m_pc^2/h\right)+i\left(m_\nu c^2/h\right)\frac{m_\nu}{m_p}\frac{i\left(j^2+1.7071j-1.10355-(j.5303+.8269)N\right)}{(2\alpha)^2}\right)\right)t \qquad (19.23)$$

$=\exp i\left(\left(\left(m_pc^2/h\right)+i\Delta\right)\right)t$ giving hyperon, Kaon, ±π decay times.

The second term Δ is also the excess mass above the proton mass.

For neutrons (939Mev) the excess mass above the proton mass (938Mev) is $m_p/1000$ and $R_H \rightarrow 1000R_H$, $\Delta \rightarrow \Delta'$

$$E^2 = m_p^2 + \frac{1}{1000^2}\frac{1}{(1000R_H)^2}\left(j^2 + 1.7071j - 1.10355 - (j.5303 + .8269)N\right)$$

gives the neutron decay time.

For m_μ muons j=½, N=0 and the excess mass is $m_p/8.87 \equiv m_\mu$.

$$E^2 = m_p^2 + \frac{1}{8.87^2}\frac{1}{(8.87R_H)^2}\left(j^2 + 1.7071j - 1.10355 - (j.5303 + .8269)N\right)$$

gives time for muon m_μ decay.

For π^o decay time $m_\nu \rightarrow m_e$ (E&M decay) along with $8.87 \rightarrow 7 = m_p/m_{\pi o}$ in the above equation.

For resonances $m_\nu \rightarrow m_e$ (E&M decay) in 19.23 gives time of decay.
Note the second term here contains a ii=-1 and so it is a exponential decay term e^{-Et} with .693/E=t the "half life".
Thus we get π^o, $\pm\pi$, K mesons and hyperon, muon, neutron, resonance half lives from (these modifications of) equation 19.23.

19.7 CASE 2 Excited State F, charge=0.

Recall from 19.4 that case 1 implies $E_q \rightarrow m$ in case 2 (in 19.4). Also
$1/g_{rr} \approx 1 - k_H/k_H + \varepsilon = \varepsilon$ for $-\varepsilon$. Net charge=Zero. Thus let $R = k_H + r$, $r << R$, $r' = k_H\varepsilon + r$

$$\left[\left(\frac{dt}{ds}\sqrt{g_{oo}}m\right) + m\right]F - \hbar c\left(\sqrt{g_{rr}}\frac{d}{dr} + \frac{j + 3/2}{r}\right)f = 0$$

$$\left[\left(\frac{dt}{ds}\sqrt{g_{00}}m\right) - m\right]f + \hbar c\left(\sqrt{g_{rr}}\frac{d}{dr} - \frac{j - 1/2}{r}\right)F = 0$$

$$\sqrt{g_{rr}} = 1/\sqrt{1 - k_H/R + \varepsilon} \approx \frac{\sqrt{R}}{\sqrt{R - k_H + R\varepsilon}} =$$

$$\frac{\sqrt{k_H + r}}{\sqrt{k_H + r - k_H + (k_H + r)\varepsilon}} \approx \frac{\sqrt{k_H + r}}{\sqrt{k_H\varepsilon + r(1 + \varepsilon)}} \approx \frac{\sqrt{k_H}}{\sqrt{k_H\varepsilon + r}} = \frac{\sqrt{k_H}}{\sqrt{r'}}$$

Also $(dt/ds)\sqrt{g_{oo}} \rightarrow E$ in the Dirac equation 18.1. Therefore equation 19.1 reads: $r' = k_H\varepsilon + r$

$$[E + m]F - \hbar c\left(\sqrt{\frac{k_H}{r'}}\frac{d}{dr} + \frac{j + \frac{3}{2}}{k_H + r}\right)f =$$

$$[E + m]F - \hbar c\left(\sqrt{k_H/r'}\frac{d}{dr} + \frac{j + \frac{3}{2}}{k_H + r}\right)f = 0$$

$$[E+m]F-\hbar c\left(\sqrt{k_H/r'}\frac{d}{dr'}+\left(1-\frac{r}{k_H}\right)\frac{j+\frac{3}{2}}{k_H}\right)f=0$$ and

$$[E-m]f+\hbar c\left(\sqrt{\frac{k_H}{r'}}\frac{d}{dr}-\left(1-\frac{r}{k_H}\right)\frac{j-\frac{1}{2}}{k_H}\right)F=0$$ Thus

.

$$f=-\frac{\hbar c}{|E-m|}\left(\sqrt{k_H/r'}\frac{d}{dr'}-\left(1-\frac{r}{k_H}\right)\frac{j-\frac{1}{2}}{k_H}\right)F$$ Therefore

$$[E+m]F-\hbar c\left(\sqrt{k_H/r'}\frac{d}{dr'}+\left(1-\frac{r}{k_H}\right)\frac{j+\frac{3}{2}}{k_H}\right)f=0$$ Using $r=r'+k_H\varepsilon$

$$[E+m]F-\hbar c\left(\sqrt{k_H/r'}\frac{d}{dr'}+(1-((r'-\varepsilon k_H)/k_H))\frac{j+\frac{3}{2}}{k_H}\right)\frac{-\hbar c}{|E-m|}\left(\sqrt{k_H/r'}\frac{d}{dr'}-(1-((r'-\varepsilon k_H)/k_H))\frac{j-\frac{1}{2}}{k_H}\right)F=0$$

Multiplying both sides by $|E-m_p|$ we obtain:

$$\frac{E^2-m^2}{(\hbar c)^2}F+\left(\sqrt{k_H/r'}\frac{d}{dr'}+\left(1-\frac{(r'-k_H\varepsilon)}{k_H}\right)\frac{j+\frac{3}{2}}{k_H}\right)\left(\sqrt{k_H/r'}\frac{d}{dr'}-\left(1-\frac{r'-k_H\varepsilon}{k_H}\right)\frac{j-\frac{1}{2}}{k_H}\right)F=0$$

$$\frac{E^2-m^2}{(\hbar c)^2}\text{-}\left((1+2\varepsilon)\frac{(J+3/2)(J-1/2)}{k_H^2}\right)F+\sqrt{k_H/r'}\frac{J-1/2}{k_H^2}+(1+\varepsilon)\frac{(J+3/2)}{k_H}\sqrt{\frac{k_H}{r'}}\frac{d}{dr'}+$$
$$\left(\frac{k_H}{r'}\frac{d^2}{dr^2}-\frac{1}{2}\frac{k_H}{r^{4/2}}\frac{d}{dr'}\right)F=0.$$ Multiplying both sides by r'^2 we obtain:

$$\left(\left[\frac{E^2-m^2}{(\hbar c)^2}r'^2\right]-(1+2\varepsilon)(J+3/2)(J-1/2)\frac{r'^2}{k_H^2}\right)F+(1+\varepsilon)\frac{(J+3/2)}{\sqrt{k_H}}r^{3/2}\frac{d}{dr'}+$$
$$\left[\left(r'k_H\frac{d^2}{dr'^2}\right)-\frac{1}{2}k_H\frac{d}{dr'}-\frac{\sqrt{k_H}r'^{3/2}}{k_H^2}(J-1/2)\right]F=$$

Defining $r'\equiv r^2$ and doing the derivatives in the new variable:

$$\frac{dF}{dr^2}=\frac{dF}{dr}\frac{dr}{dr^2}=\frac{1}{2r}\frac{dF}{dr}$$ and

$$\frac{d^2F}{dr^2}=\frac{1}{2r}\frac{d}{dr}\left(\frac{1}{2r}\frac{dF}{dr}\right)=\frac{1}{2r}\left(-\frac{1}{2r^2}\frac{dF}{dr}\right)+\frac{1}{2r}\frac{1}{2r}\frac{d^2F}{dr^2}=$$

$-\frac{1}{4r^3}\frac{dF}{dr}+\frac{1}{4r^2}\frac{d^2F}{dr^2}$ Substituting these expressions for the derivatives in:

$$\left[\frac{E^2-m^2}{(\hbar c)^2}r^4-(1+\varepsilon)(J+3/2)(J-1/2)\frac{r^4}{k_H^2}\right]F+$$

$$\left[\frac{k_H}{4}\frac{d^2}{dr'^2}-\frac{1}{4r}k_H\frac{d}{dr'}\right]F-\frac{\sqrt{k_H}\,r'^3\,(J-1/2)}{k_H^2}F+\frac{r^3(1+\varepsilon)(J+3/2)}{k_H}\frac{\sqrt{k_H}}{2r}\frac{d}{dr'}F+$$

$$-\frac{k_H}{4r}\frac{d}{dr}F=$$

$$\left[\left(\frac{E^2-m^2}{(\hbar c)^2}\right)-(1+2\varepsilon)\frac{(J+3/2)(J-1/2)}{k_H^2}\right]\sum a_n r^{n+4}+\frac{k_H}{4}\sum(n-1)na_n r^{n-2}-$$

$$\frac{k_H}{4}\sum na_n r^{n-2}-\frac{\sqrt{k_H}}{k_H^2}\left(J-\frac{1}{2}\right)\sum a_n r^{n+3}+(1+\varepsilon)\frac{(J+3/2)}{2\sqrt{k_H}}\sum na_n r^{n+1}-\frac{k_H}{4}\sum na_n r^{n-2}=$$

$$\left(\frac{E^2-m^2}{(\hbar c)^2}-(1+2\varepsilon)\frac{(J+3/2)(J-1/2)}{k_H^2}\right)\sum a_{n-4}r^n+\frac{k_H}{4}\sum(n+1)(n+2)a_{n+2}r^n-$$

$$\frac{k_H}{4}\sum\ (n+2)a_{n+2}r^n-\frac{\sqrt{k_H}}{k_H^2}\left(J-\frac{1}{2}\right)\sum a_{n-3}r^n\quad+(1+\varepsilon)\frac{(J+3/2)}{2\sqrt{k_H}}\sum(n+1)a_{n-2}r^n-$$

$$\frac{k_H}{4}\sum(n+2)a_{n+2}r^n=0.$$

Combining terms noting simplification due to combining the a_{n+2} terms

$$\left(\frac{E^2-m^2}{(\hbar c)^2}-(1+2\varepsilon)\frac{(J+3/2)(J-1/2)}{k_H^2}\right)\sum a_{n-4}r^n+\frac{k_H}{4}\sum(n-1)(n+1)a_{n+2}r^n+$$

$$\frac{\sqrt{k_n}}{k_H^3}(J-1/2)\sum\ a_{n-3}r^n+(1+\varepsilon)\frac{J+3/2}{2\sqrt{k_H}}\sum(n-1)a_{n-1}r^n=0$$

Next we write the individual eigenfunctions as:

$$\left(\frac{E^2-m^2}{(\hbar c)^2}-(1+2\varepsilon)\frac{(J+3/2)(J-1/2)}{k_H^2}\right)\sum a_{n-4}r^n=0.$$

Thus since these series terms add to zero:

$$E=\sqrt{m^2+(\hbar c)^2(1+2\varepsilon)\frac{(J+3/2)(J-1/2)}{k_H^2}}\qquad(19.24)$$

$(1+\varepsilon)\frac{J+3/2}{2\sqrt{k_H}}\sum(n-1)a_{n-2}r^n=0.$ Here $r'=r^2$ so $r^{-2/2}=r'^{-1}=\sqrt{r}^{-2}$

$$\frac{k_H}{4}\sum(n^2-1)a_{n+2}r^n=0\qquad(19.25)$$

- $\frac{\sqrt{k_H}}{k_H^2}(J-1/2)\sum a_{n-3}r^n = 0$ (19.26)

J=1/2 with N=1 solves the indicial equation implied by 19.24-19.26. Recall from 19.4 that m=proton in this case (case 2). The energy in 19.24 is then that of a neutral particle (q=0) with the mass of the neutron so E =E_q =m=m_N. See equation 19.23b for neutron lifetime and 2P3/2 for neutron spherical harmonic state, section 20.3) But in case 2 and equation 19.23 then the previously derived charged spin ½ hadrons m_Σ, m_Ξ can also be put back into the Dirac equations for 'm'(instead of the proton). Thus the charged, m_Σ, m_Ξ from equation 19.14 can be put into the "m" in 19.24 which gives the **neutral** E=m=m_N, m_Ξ.. m_Σ has a N=1/2 and so does not satisfy the above equations and so does not exhibit a stable neutral Σ. Recall the Ω^- (which is J=3/2) is not J= ½ so doesn't have a neutral counterpart as does the proton and these other J= ½ hyperons.

Recall the iterated Dirac equation is the Klein Gordon (in χ with J=0) equation eigenstate transitions.

J=0, q=0 Case 2

Recall J=0 is allowed in every case.

m=1 proton, j=0 in equation 19.24 means K Long. Equation 19.23 gives K long mass eigenvalue: 1+(0+3/2)(0-1/2)/1=1/4. Thus √.25 =.5. Thus .5X938X1.06=497 MeV=K_{long}. Note case 2 is zero charge and note also from section 19.8 that the Strangeness=2|√.5|=2*.707 ≈1 as in strangeness equation 19.34 below.

m≈1 for Neutron then in 19.24 we have K short, if m=m_Ξ and J=0 then D^o Long.

If m=m_Ξ j=0, and neutral then 19.24 gives D^o Short.

19.8 CASE 3 m=0, so ψ_L, f state, charge=0 (lower case of equation 19.5).

In case 3 there is no central force therefore N=0 and j=½ in f. This is the m=0 left handed doublet case of equation 12.8. Let R=k_H-r, r<<R for stability we can write:

$$\sqrt{g_{rr}} = 1/\sqrt{1+k_H/R+\varepsilon} \approx \frac{\sqrt{R}}{\sqrt{R+k_H+R\varepsilon}} =$$

$$\frac{\sqrt{k_H-r}}{\sqrt{k_H-r+k_H+(k_H-r)\varepsilon}} = \frac{\sqrt{k_H-r}}{\sqrt{k_H(2+\varepsilon)-r(1+\varepsilon)}} = \frac{(1-\varepsilon/2)}{\sqrt{2}}\left(\frac{1-\frac{r}{2k_H}+\frac{r^2}{8k_H^2}+..}{1-\frac{r}{4k_H}+\frac{r^2}{16k_H^2}+..}\right) =$$

((1-ε/2)/√2) $\left(1-\frac{r}{4k_H}+\frac{3r^2}{32k_H^2}-..\right) \approx \frac{1-\frac{r}{4k_H}}{\sqrt{2}}$

Therefore equation 19.1 reads:

$$[E+m_p]F - \hbar c\left((1-r/k_H 4)\frac{d}{\sqrt{2}dr}+\frac{j+\frac{3}{2}}{k_H-r}\right)f = 0$$

$$\left[E+m_p\right]F-\hbar c\left(\left(1-r/k_H 4\right)\frac{d}{\sqrt{2}dr}+\left(1+r/k_H\right)(j+\frac{3}{2})/k_H\right)f=0\,;$$

$$\left[E-m_p\right]f+\hbar c\left(\sqrt{g_{rr}}\,\frac{d}{dr}-\frac{j-\frac{1}{2}}{r}\right)F=0 \qquad (19.27)$$

From the above equation 19.27 if (and j= ½) m_p=0 then

$$\left[E-m_p\right]f+\hbar c\left(\left(1-r/k_H 4\right)\frac{d}{\sqrt{2}dr}-\left(1+r/k_H\right)\left(j-\frac{1}{2}\right)/k_H\right)F=0$$

Therefore (with j= ½) from equation 19.27 for small r. In any case:

$$F=\hbar c\frac{\hbar c}{\left[E+m_p\right]}\left(\left(1-r/k_H 4\right)\frac{d}{\sqrt{2}dr}+\left(1+r/k_H\right)\left(j+\frac{3}{2}\right)/k_H\right)f$$

$$\left[E+m_p\right]F-\hbar c\left(\left(1-r/k_H 4\right)\frac{d}{\sqrt{2}dr}+\frac{j+\frac{3}{2}}{k_H-r}\right)f=0$$

Solving for *f* and substituting back in 19.27

$$\left[E-m_p\right]f+\hbar c\left(\left(1-r/k_H 4\right)\frac{d}{\sqrt{2}dr}-\left(1+r/k_H\right)(j-.5)/k_H\right)\bullet$$

$$\frac{\hbar c}{E+m_p}\left(\left(1-r/k_H 4\right)\frac{d}{\sqrt{2}dr}+\left(1+r/k_H\right)(j+\frac{3}{2})/k_H\right)F=\left[E-m_p\right]f+$$

$$\frac{(\hbar c)^2}{\left(E+m_p\right)\sqrt{2}}\left(-\left(1-r/4k_H\right)\frac{d}{k_H 4\sqrt{2}dr}+\left(1-r/4k_H\right)^2\frac{d^2}{\sqrt{2}dr^2}+\left(1-r/4k_H\right)(j+\frac{3}{2})/k_H^2\right)f$$

$$+\frac{(\hbar c)^2}{E+m_p}\left(-\left(1+3r/k_H 4\right)(j-.5)\frac{d}{\sqrt{2}k_H dr}-\left(1+r/k_H\right)^2\left(j+1.5\right)(j-\frac{1}{2})/k_H^2\right)f=$$

$$\left(\left[E-m_p\right]+\left[\frac{(\hbar c)^2}{\left(E+m_p\right)}\left(-\left(j+\frac{3}{2}\right)(j-\frac{1}{2})/k_H^2+(j+\frac{3}{2})/\sqrt{2}k_H^2\right)\right]\right)f+$$

$$\frac{(\hbar c)^2}{\left(E+m_p\right)\sqrt{2}}\left(2\sqrt{2}\left(j+\frac{3}{2}\right)(j-\frac{1}{2})/k_H^3-(j+\frac{3}{2})/4k_H^3\right)rf+$$

$$\frac{(\hbar c)^2}{\left(E+m_p\right)\sqrt{2}}\left(-\frac{1}{k_H 4\sqrt{2}}+\left(j-\frac{1}{2}\right)\frac{1}{k_H}\right)\frac{df}{dr}+$$

$$\frac{(\hbar c)^2}{\left(E+m_p\right)\sqrt{2}}\left(\frac{1}{k_H^2 16\sqrt{2}}-\left(j-\frac{1}{2}\right)\frac{3}{4k_H^2}\right)r\frac{df}{dr}+$$

$$\frac{(\hbar c)^2}{\left(E+m_p\right)\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)\frac{d^2 f}{dr^2}+$$

$$\frac{(\hbar c)^2}{(E+m_p)\sqrt{2}}\left(\frac{-1}{2\sqrt{2}k_H}\right)r\frac{d^2 f}{dr^2}$$

Next substitute in F= $\sum_n a_n r^n$

$$\sum_M^N\left(\left[E-m_p\right]+\left[\frac{(\hbar c)^2}{(E+m_p)}\left(-\left(j+\frac{3}{2}\right)(j-\frac{1}{2})/k_H^2+(j+\frac{3}{2})/\sqrt{2}k_H^2\right)\right]\right)a_n r^n +$$

$$\sum_M^N\frac{(\hbar c)^2}{(E+m_p)\sqrt{2}}\left(2\sqrt{2}\left(j+\frac{3}{2}\right)(j-\frac{1}{2})/k_H^3-(j+\frac{3}{2})/4k_H^3\right)a_{n-1}r^n + \quad (19.28)$$

$$\sum_M^N\frac{(\hbar c)^2}{(E+m_p)\sqrt{2}}\left(-\frac{1}{k_H 4\sqrt{2}}+\left(j-\frac{1}{2}\right)\frac{1}{k_H}\right)(n+1)a_{n+1}r^n + \quad (19.29)$$

$$\sum_M^N\frac{(\hbar c)^2}{(E+m_p)\sqrt{2}}\left(\frac{1}{k_H^2 16\sqrt{2}}-\left(j-\frac{1}{2}\right)\frac{3}{4k_H^2}\right)na_n r^n + \quad (19.30)$$

$$\sum_M^N\frac{(\hbar c)^2}{(E+m_p)\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)(n+2)(n+1)a_{n+2}r^n + \quad (19.31)$$

$$\sum_M^N\frac{(\hbar c)^2}{(E+m_p)\sqrt{2}}\left(\frac{-1}{2\sqrt{2}k_H}\right)(n+1)na_{n+1}r^n = 0 \quad (19.32)$$

We now take 19.27 and 19.30 together and 19.29 and 19.32 together (since they have the same a_n). Thus there are 4 independent series (with 19.28 and 10.31) here. The equation 19.27 and 19.30 nth terms give:

$$\left(\left[E+m_p\right]+\left[\frac{(\hbar c)^2}{(E-m_p)}\left(-\left(j+\frac{3}{2}\right)(j-\frac{1}{2})/k_H^2+(j+\frac{3}{2})/\sqrt{2}k_H^2\right)\right]\right)+$$

$$\frac{(\hbar c)^2}{(E-m_p)\sqrt{2}}\left(\frac{1}{k_H^2 16\sqrt{2}}-\left(j-\frac{1}{2}\right)\frac{3}{4k_H^2}\right)n,$$

At some value of n=N we have for a solution

$$(E^2-m_p^2)+\left(\left[\left[\left(-\left(j+\frac{3}{2}\right)(j-\frac{1}{2})/k_H^2+(j+\frac{3}{2})/\sqrt{2}k_H^2\right)\right]\right]\right)+\frac{1}{\sqrt{2}}\left(\frac{1}{k_H^2 16\sqrt{2}}-\left(j-\frac{1}{2}\right)\frac{3}{4k_H^2}\right)N=0$$

therefore rearranging:

$$E=\sqrt{m_p^2+\frac{1}{k_H^2}((j+3/2)(j-1/2)+.7071j+1.0607+(.0156-(j-.5).5303)N)} \quad (19.33)$$

Recall from the equation 19.4 'f' case that we have m_p=m=0, and zero charge therefore no central force thus N=0 in $f\propto r^o$ in equation 18.1. Therefore since there is small r and dr^0/dt=d1/dt=0 in the equations just above equation 19.27 along with 19.33 then the 19.27-19.32 equations add to zero and thus are solved. Also the j=3/2 (so L=1) case is not allowed since that requires a central force to give L≠0, j= ½ and of course j=0 is allowed here. Thus
N= **0**, j= ½, m=0 then from 19.33 we have E=1115.8 Mev Λ_o

N=0, j=0, η mass and also gives m=.56 (with m=0) in 19.33 used in gyromagnetic ratio calculation for f. Recall ε=.08 (with m=0) for F in 19.14. This is the nontrivial f state zero point energy for r<k since Ψ=ψ+χ from our observability definition. Note Kaons then give no strange bound states because this mass is real (in contrast to the imaginary pion mass in 19.22).

19.9 Strangeness

Recall that in 19.14 (which applies to Case 1 and Case 2) the energy is $E^2=m_o^2+(j^2+1.7071j-1.10355-(j(.53033))+.7642))N)/k_H^2$. Now m_o^2 and E is conserved (m_o is a constant) here and thus it appears that energy conservation implies that the square root of $j^2+1.7071j-1.10355)-(j.5303+.7642)N \equiv S$ must be conserved. Therefore $E^2=m^2+S^2$ then and "S" is conserved for the charged core states and thus for the neutrals given that in section 19.8 that $E_q \rightarrow m$ then (for f state m=0 we also have S≈E for Λ). We could also write $E^2=m^2+C^2$ for the next 2P state eigenstates (call C charm if you want) which would also have there own associated production (since <|> not zero). Thus, as an example, normalizing to a factor of 2X:

$2XSQR[(.5(.5303)+.7642)(0)]=0=S_{nucleon}$, $2XSQR|[(.5(.5303)+.7642)(-1)|\approx 2=S_{\Xi}$,
$2XSQR|[.5(.5303+.7642)(-½)]|\approx 1=S_{\Sigma}$, $2X\,SQR|[(1.5^2+1.7071(1.5)-1.10355-(1.5(.5303+.7642)))(-1)]|\approx 3=S_{\Omega}$. (19.34)

Strangeness is only an approximate conservation law in the examples in 19.34 but there is enough conservation at least for the "associated production" and we have not yet included the weak interaction here. This is a **direct derivation of strangeness**, instead of just having postulated it as it is in the standard model and QCD. Strangeness isn't strange anymore.

Charm, bottom, top: In chapter 19 equation assuming hard spherical shell. We obtain other (less stable, resonances) particle groups using equation 19.5 by taking the quadratic approximation of g_{rr} (i.e., include the $(3/32)(r/k_H)^2$ term in 19.5) instead of just the linear approximation we used above. Recall that the perturbative $(3/32)(r/k_H)^2$ term had to be included since it gave a ≈20Mev correction to the hyperon masses.

Chapter 20

$r \approx r_H$ Application: 2P3/2 Half Integer Spherical Harmonics Solutions. This is a continuation of Chapter 19

20.1 "r" for Maximum F=-dE/dr

Recall the electron ($\Delta\varepsilon=m_e$) horizon in $r_H=e^2/2m_ec^2$ ~1fermi For the covalent bonds at $r=r_H$ the fields from these section 19.6 two covalently bonded ±e s cancel at large distance making this a localized force at 1fermi analogous to the Van der Waals force. Recall $E=1/\sqrt{g_{00}}$ with $g_{oo}= 1-r_H/r$ so that this force is stronger than E&M at the $r=r_H$ singularity in F=-dE/dr at $r=r_H$.

From section 19.5 we saw that substituting J=1, N=-1 into equation 19.33 gives the deuteron mass energy where the deuteron force (found from the non m_p part of the energy in equation 19.3) is the paradigm for the nuclear force. r is scaled higher because of $1+\varepsilon \rightarrow 2$ in $d/(\sqrt{(1+\varepsilon)}dr)$ at the earliest era just after the big bang (see equation 18.3) so dr scales $\sqrt{2}$ larger so k_H scales $\sqrt{2}$ larger. But since k_H is squared in the denominator of equation 19.33 then E appears lower so dE/dr is smaller by a factor of 2 for given dr. Thus given larger ε at about 12 billion years ago the nuclear force dE/dr at that time was smaller relative to E&M forces by a factor of 2 than it is now. Also recall that iron was the termination nucleon number for fusion energy release; where this number came from the ratio of nuclear to E&M force magnitude. Thus 12 billion years ago for example (when ε was near 1) fusion released energy only up to silicon range (26/2 protons) of nucleon number instead of the iron range. Thus at that era supernova produced silicon and carbon instead of iron and therefore thick interstellar dust resulted. Therefore that era was dusty giving a strong infrared background around galaxies.

The triplet scattering length $2e^2/m_ec^2 = a_t=5.38$ $F=r_H$. This corresponds to the triplet scattering length of the Deuteron. Recall the Deuteron is in L=1, $2P_{3/2}$ state which is observed from the outside to be S=1. The triplet state is associated with ***nuclear force*** scattering.

20.2 r_H is Hard Shell Van Der Waals (liquid) Equation of State

Again from the g_{oo} in equation 19.1 the clocks slow down and stop as r_H (since the metric time component g_{oo} is then small) is approached making this a hard sphere scattering situation and so a Van der Waals type equation of state. Recall the Van der Waals equation of state implies a liquid at certain temperatures and pressures because it includes the effects of a hard shell. Those 100 GeV BNL gold-gold collisions should have then given rise to a liquid equation of state as well according to this theory. But the standard model implies a *point like* quark gluon plasma equation of state. So what was found at BNL? BNL found the liquid equation of state making the standard model bogus in contrast to this one.

20.3 Overview of $2P_{3/2}$ Solutions to Equation 1.9 (the New Dirac equation) at $r \approx r_H$ in the Context of the Equivalence Principle (single charge e) Implication

Allowing this *single* charge '*e*' to move near and inside that stable singularity radius $r \approx 2q^2/mc^2$ in the $\sqrt{g_{ij}}$ in this new Dirac equation (equation 1.6) as we see below makes the motion relativistic but stable requiring all the Dirac equation spherical harmonic solutions, not just the ones allowed by the Schrodinger equation. Also the next order of approximation above the hard shell for our g_{oo} horizon $r_H = 2e^2/m_e c^2$ is the harmonic oscillator $V \propto r^{+1}$ giving the *SU(3) SYMMETRY* of the three dimensional harmonic oscillator. The *+1* in the exponent of V (instead of the inverse square law-1) also reverses the sign on the exchange integral $\pm\int\psi^*_{111}(r')\psi^*_{lmn}(r'')V(r',r'')\psi_{lmn}(r')\psi_{111}(r'')d\tau = J$ designating the symmetric and antisymmetric states), making here then the $J=3/2$ state $m=-3/2$ and 3/2
(i.e., $\psi = \mathcal{Y}^{3/2}_{3/2}(\theta,\phi) + \mathcal{Y}^{-3/2}_{3/2}(\theta,\phi) = 2P_{3/2}$ eigenspinor) the first ground state that varies with azimuthal angle(baryons) above the already filled 1S (in analogy with helium) on the energy ladder instead of the expected ½ and –½ (these ½ s by the way give $2P_{1/2}$ in the $\psi^*\psi$ of the next higher P orbital slots) that vary with azimuthal angle (baryons).

Also recall the identity $(exp(i\phi)+exp(-i\phi))/2 = cos\phi$. The $\mathcal{Y}^{3/2}_{3/2}$ orbital is a $exp(i3/2\phi)$ and $Y^{-3/2}_{3/2}$ orbital is $exp(-i3/2\phi)$ and thus from the identity the summed state is $cos(3/2\phi)$ with probability density $\psi^*\psi = cos^2(3/2\phi)$, the trifolium three lobed shape. Thus there are TWO +e s giving a net charge of +2/3e in each lobe because the +electron charge 'e' is in each orbital lobe on the average only *1/3 the time* (*FRACTIONAL CHARGE*) giving the many scattering properties (such as jets) associated with the angular distribution of multiple fractional charges interior to this horizon. The lobe 'structure' *can't leave (ASSYMPTOTIC FREEDOM)* as in the Schrodinger equation case or move so is *NONRELATIVISTIC* in contrast to its rapidly moving m_e constituent.

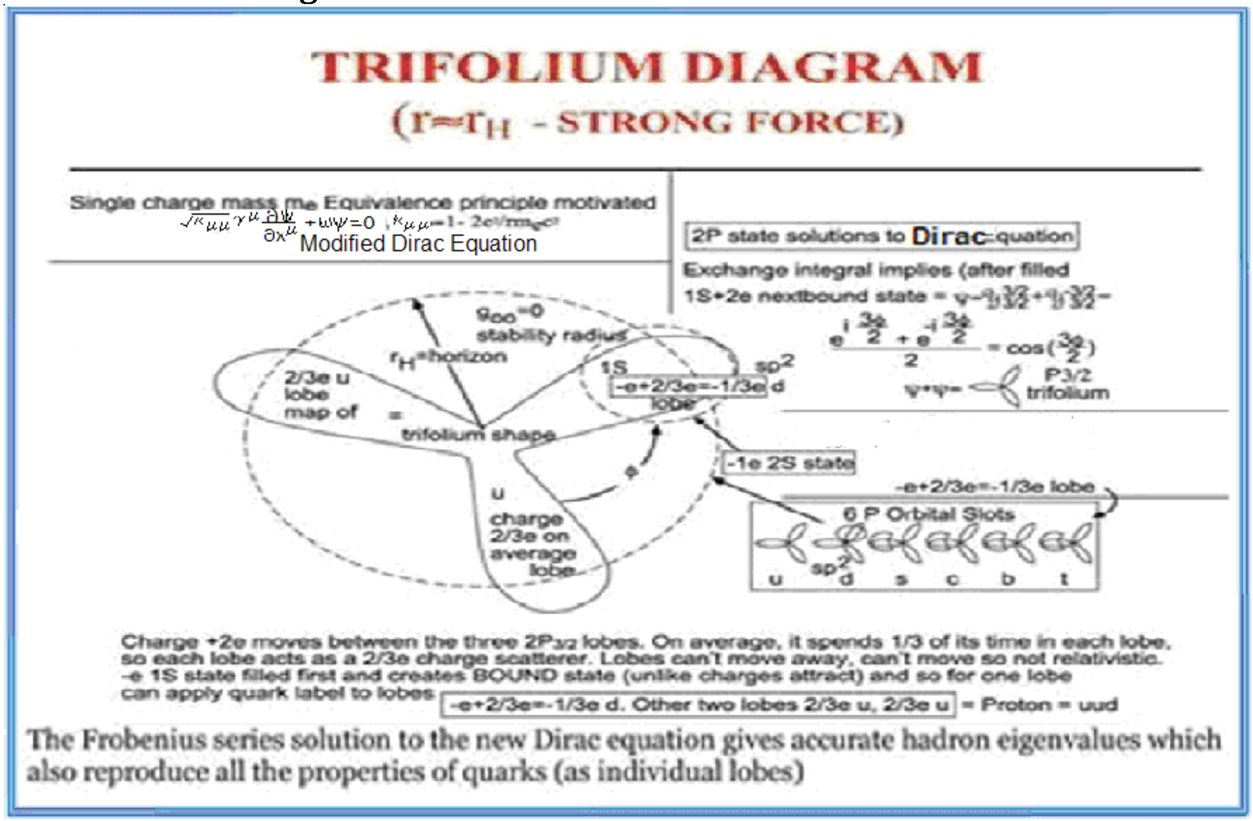
To obtain bound states we must hybridize with a *–e* (negative attracts positive). However the sp hybrid applies to a single such Y but there are two here. Thus the first 2S state –e sp^2 hybrid *-e+2/3e=**-1/3**e LOBE* is required for this to be a *BOUND* state. The rest again are those *+2/3e* lobes and thus we have the *-1/3e (d), +2/3e* (u) quark theory!! (here uud proton). sp^2 is conveniently 3 lobed also so the next –e covers another +2/3e lobe, thus making the next state a neutral particle ddu(neutron). The next 4 of the 6 P *ORBITAL SLOTS* add limacon convolutions that constitute the (in addition to these u,d) s,c,b,t *FLAVORS*. In any case we see that there is still total unit charge ±e (or 0 for this multibody system) so we don't need to postulate *COLOR* to guarantee this.

Hyperons and Mesons By Combining Integer Charge

$2P_{3/2}$ filled is 2e and gives +2/3 charge in three trifolium lobes (equivalent to two u). Take these two electrons to be in the first P state (there are three m=-1,0,+1 substates of L=1 altogether). Add (first P state) -e to one lobe to create bound state (equivalent to d), get the proton.
Add to this proton state the $2P_{1/2}$ +e and -e to get net J=1/2=l-S=1-1/2=1/2. That cancels the proton spin 1/2 but does not add extra charge. Then add yet another -e to get another d lobe and you get the neutron. Thus you get the correct spin and charge of the neutron. This explains most of the additional ~2MeV excess mass of the neutron above the proton in a very nice way: there are three added electron mass energies!!
For a π^o pion you just need -e and +e. For the +pion you need again that $2P_{1/2}$ to give zero charge but spin 1/2 and then add +e to cancel the spin 1/2 and give positive charge. Thus the pion shares

the $2P_{1/2}$ state with the hyperons, has nonzero interaction matrix with the hyperons, so it can also be that Yukawa bond exchange particle. . By exchanging this $2P_{1/2}$ state charged pion the neutron and proton interchange identities since the neutron also has a $2P_{1/2}$ state. Thus by exchanging charged pions the protons and neutrons are alternately throwing a $2P_{1/2}$ state back and forth.
So we have used up 2 out of the 6 P state electrons. Use the Frobenius series solution to hint at a third P electron state of about 1/4GeV and start building the kaons, and strange hyperons the same way.
Note that constructing the hadrons from integer charge means the neutron must have those 3 extra electrons with two of them in that $2P_{1/2}$ state. Thus we explain that ~2MeV added (above the proton mass) to get the neutron. We also have a rock solid explanation of the nuclear force: the neutron is transmitting its $2P_{1/2}$ wave function component via the charged pion, which also has this component. Notice also there is no purely 2 proton nucleus even though according to the standard theory there would be a strong nuclear force between these two protons.
There then would not be the exchange of that $2P_{1/2}$ state via the charged pion for the two proton case, so no such nuclear force is possible.
Finally we solve the problem with the new pde using a computer program, set the boundary conditions as if the Deuteron was a square well. See end of chapter 20 for the fortran program. In any case we can build the hyperons and mesons with integer charges e, don't need the fractional charges.

20.4 Trifolium Diagram



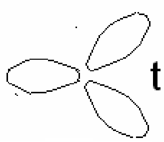
$2P_{3/2}$ solutions to Dirac equation. $\sqrt{\kappa_{\mu\mu}}\gamma^\mu \frac{\partial\Psi}{\partial x^\mu} - \omega\Psi = 0$ $\quad \kappa_{oo} = 1 - \frac{r_H}{r}$

Ultrarelativistic LS coupling fills $2P_{3/2}$ first $\quad \kappa_{oo} = \frac{1}{\kappa_{rr}}$

Stability at $r \approx r_H$ since then $\kappa_{oo} = 0$

Exchange integral implies (after filled 1S +2e) next bound state =

$$\psi = Y^{3/2}_{3/2} + Y^{-3/2}_{3/2} = \frac{e^{i\frac{3}{2}\phi} + e^{-i\frac{3}{2}\phi}}{2} = \cos\left(\frac{3}{2}\phi\right)$$

$\psi^*\psi = \cos^2\left(\frac{3}{2}\phi\right) =$  trifolium

Electron charge e spends 1/3 of its time in each lobe making each lobe (1/3)e charged on average. For two such electrons it is (2/3)e.

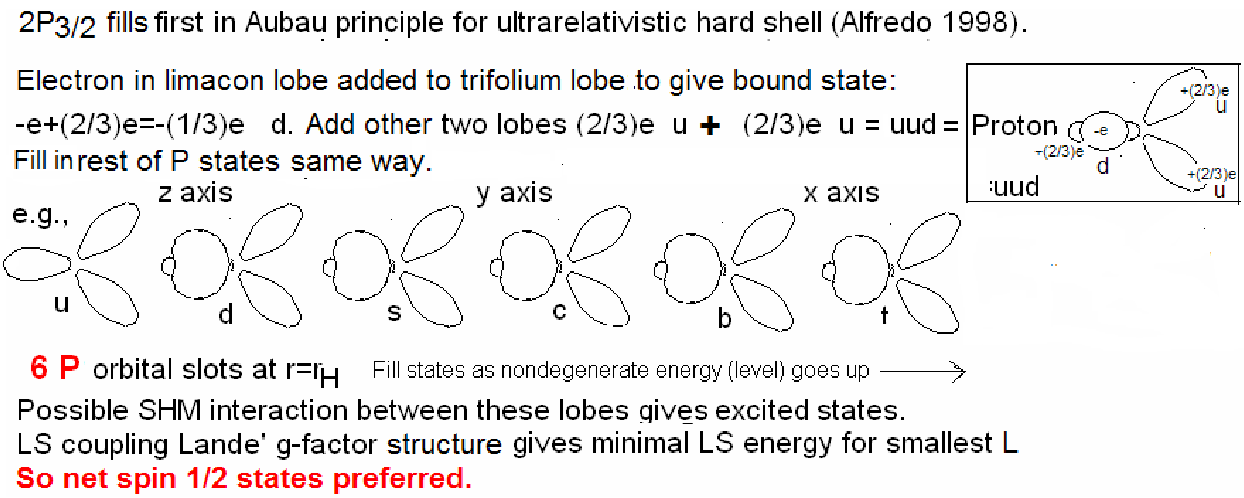
Start with 1S state filled singlet =(1/2 -1/2) = Y^0_0. Add $2P_{3/2}$ and electron -e for bound state e.g., proton.

Add to this proton $2P_{3/2}$ state -e and +e in filled $2P_{1/2}$ and a third -e with spin down to get neutron(spin1/2)

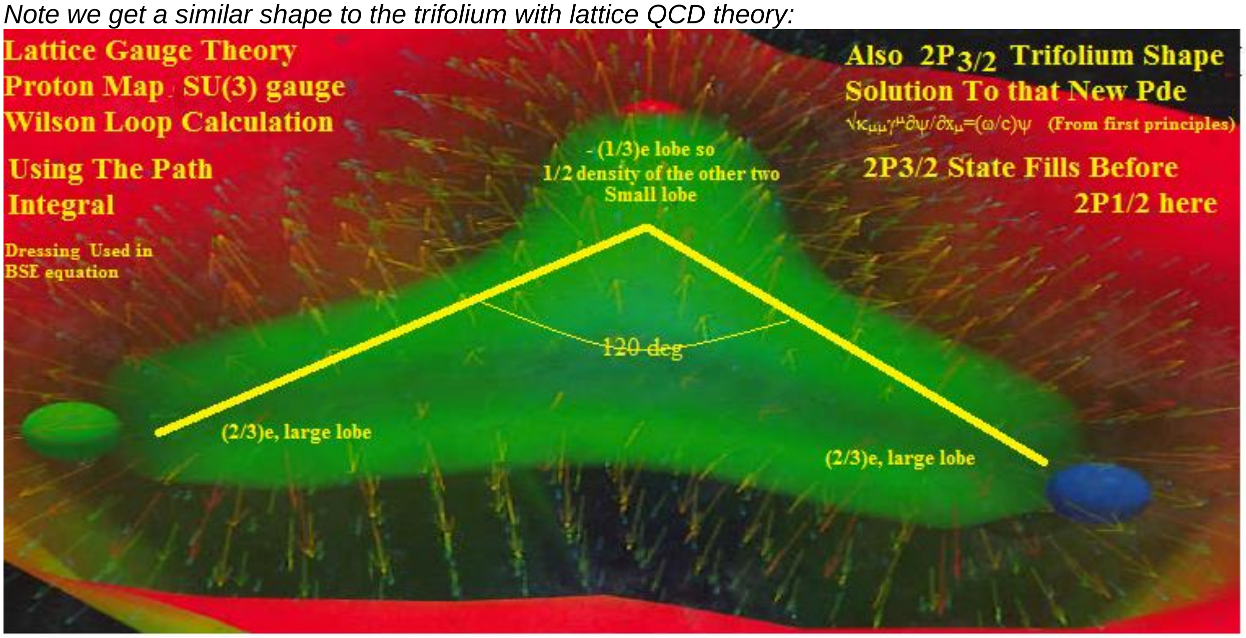
Singlet = π^0 with one of the electrons $\Delta\varepsilon$ in the first electron excited state ε(muon).

Add -e and +e in filled $2P_{1/2}$ and a third +e with down spin get π^+ (spin0)

Figure 20-1 Trifolium diagram



Note we get a similar shape to the trifolium with lattice QCD theory:



20.4 Summary of $2P_{3/2}$ at $r=r_H$ results

Short of using the Frobenius method (and variational methods for many particle systems) for exact answers we can still understand the origin of at least the first rungs on this energy ladder. For example the two (antinode) electrons for L=0 are constrained to each move along the diameter $\lambda=2R_e$ *where* $R_e=2e^2/m_ec^2$ given that $g_{oo}=1-2e^2/rm_ec^2$. Substituting this (L=0) λ into $\omega=2\pi c/\lambda$ and then ω into (equation 1.9) value $m=h\omega/c^2$ results in the $mc^2 \approx$ 1S energy~.11GeV (also approximating the slightly higher E_π for the equation 2 Klein Gordon L=0 S state solutions), far larger than m_e for *non*rotating 1S state so having ultrarelativistic transverse motion. After the 1S state is filled the *rotating* L=1, 2P Dirac equation (equation 1.9) state then must also be relativistic and rotational velocity component contracted to

$$r_H=R_e\int_0^{\pi/2}\left(\sqrt{1-c^2\sin^2\theta/c^2}\right)\sin\theta d\theta=R_e/2. \qquad (20.2)$$

By this same Lorentz factor there is a X2 mass increase so a net 4X decrease in $2e^2/m_ec^2 \rightarrow 2e^2/4m_ec^2 = 1.4F$, close to the proton effective interaction "radius". But there are at least two such λ on the trifolium implying, with the previous 1S reduced mass energy, $m=1S+2h\omega/c^2 \approx .94$GeV ($=m_p=(M_{2S}+M_{1S}+m_e)/2$ since 2P and reduced mass 2S belong to the same principle quantum number with nearly equal ground state mass m eigenvalues again implying that the Dirac L=0 S state motion is entirely transverse with a huge Lorentz contraction (causing isolated S states to be the "point" source leptons).

In summary (For $R_e=e^2/m_ec^2$) the 'big' $2R_e=D_e$ holds the L=0 two antinodes for the 1S muon λ. But for L=1 rotation the 2P diameter gets Lorentz contracted (along with the 2X mass increase in the denominator) to the 'little' $D_e/4$ holding the proton λs.

Also at the radii where $g_{oo}\approx 0$ we note that $2P_{3/2}$ has highest probability density (in contrast to higher orbitals) and in the ds^2 metric formula (equation 5.1) we see that the time component $g_{oo}dt^2$ is small $g_{oo}\approx 0$ so clocks slow down therefore there is (proton) *STABILITY*. Note this singularity is on a horizon surface that is farther out, at the $2P_{3/2}$ state r_H=1 .4 Fermi distance making a far larger force P state than for point source E&M (isolated S states) out at these radii ($F_\pi\equiv dE/dr\approx E_\pi/r_H\approx$ 100Xelectrostatic force near r_H). This force, just as in 2P orbital atomic physics, is exhibited also as a short range *COVALENT BOND BETWEEN* these *multiparticle P STATES*, giving dinucleon (deuteron) states with our 2P state of L=1 with these interweaved covalent bond shared electron lobes being the equivalent of the *two* lobed (force exchange) pions which also then have shared lobe constituents of X1/3e fractional charge as well. Note the condensed matter state of these $P_{3/2}$ objects contain these *hard shell* g_{oo} horizons r_H implying a Van der Waals EQUATION OF STATE *LIQUID* (e.g., in gold-gold nucleus collisions**)**, not a gas. Thus we explain the:

1) Fractional (multiples of *1/3e*) charge of quarks
2) Assymptotic freedom in hadrons of quarks
3) Unit *total* quark (summed) charge e in hadrons
4) Nonrelativistic quark velocity
5) Stability of proton
6) Large quark masses
7) Correct proton size
8) Short range covalent bonding of correct strength and L=1.
Etc.

There is no other way to explain this multiplicity of quark properties in such a simple way. Thus quarks really don't exist at all, all there is here is that single (new Dirac equation, equation 1.9) electron moving in that potential, just names for the first excited state $2P_{3/2}$ half integer spherical harmonic lobes. Thus the core (Dirac) equation (our equation 1.9) *is what works here*. Most importantly this result is suggestive of the possibility that we can also just solve that equation 1.9 (Dirac equation) for hadron eigenvalues, giving correct J and E_n for example, using the Frobenius series termination method.

So just solve that single charge equation 1.9 for its ψ! Therefore we don't have to postulate all this (in points 1-8) and we don't need the Yang Mill's theory, colors, flavors, gluons, QCD, SU(3) gauges, etc.. You have a beautiful Ockam's razor type simplification of the physics.

In fact a recent paper has come out interpreting the highest energy LEP2 scattering data in just this 'single charge' way (the charges would then appear stationary, not smeared out into these lobes then), a unit charge e moving inside the hadron was the simplest way to interpret the ($E_{incident}$>>$E_{restmass}$) data.

20.6 P Wave Scattering and Jets In 100GeV Gold-Gold Collisions

Let <A'| represent the outgoing scattering wave immediately after a incident plane wave scatters off V. Let |A> be the $2P_{3/2}$ hyperon state for $r=r_H$ having the V. Thus at $r=r_H$ V itself will have the $2P_{3/2}$*$2P_{3/2}$ =ψ*ψ trifolium shape and thus commute with |A> since they constitute the same structure ($2P_{3/2}$ commutes with itself). So since V commutes with |A> **then <A' also is a $2P_{3/2}$ state** or we have <A'|V|A>=0 and so no scattering into such states. Thus a type of 'P wave scattering' results from an incident plane wave. Thus we explain the origin of the 'jets' that are otherwise ascribed to scattering off quarks.

Note that when the mean free path d during the interaction time is very short ($d<<(1/3)\ 2\pi r_H$) there is no more smearing between the $2P_{3/2}$ lobes and we have scattering off of independent point particles and the $2P_{3/2}$ state ceases to be relevant in the scattering and so the jets disappear. (jet quenching). Thus at extremely high energy the scattering is from charge e (not 1/3e) again and there are no more jets above top energy. LEP actually observed this effect just before it was shut down.

20.7 Charge Independence Of The Strong Interaction

It is well known that the strong interaction is approximately the same magnitude between Neutron-Proton, Neutron-Neutron and Proton-Proton pairs and thus is 'charge independent'. Also note our theory deals with electrons only which only has charge dependence if certain QM effects are ignored. But recall the orthogonality of S and P states as in <S|P>=0, <S|S>=1, <P|P>=1 given all the superscript and subscript substates (e.g.,S and m) are the same as well in the bra and kets. The ordinary nuclear interaction here is due to a covalent bond (sharing electrons) which is also a very strong interaction (bond) at $r=r_H$ and is dependent on the spin S and m state and not so much on the sign of the charge. Thus these QM (valence, spin) effects are very strong at $r=r_H$. Thus the charge independence of the strong interaction is really an S state *in*dependence and $2P_{3/2}$ state dependence at $r=r_H$ of a $2P_{3/2}$ structure interacting with an S state.

20.8 Summary of Quark Properties From $2P_{3/2}$ at $r=r_H$

From the atomic physics analogy (and for example the Oxygen 3 lobes with 2 occupied by Hs in water) that 3 lobed ψ*ψ hybridization still exists even for $2P_{1/2}$. Note also that the 3 lobed character (hence the 1/3, 2/3 e charges) does not exist past the highest 2P state for at least some states (e.g., 3D states), thus past the top at ~140 GeV.

These single sp^2 electrons do not annihilate with the paired positrons because they are in different states and the paired state is filled. The filled singlet positron electron (E&M) vacuum state and the free electron do not annihilate for the same reason.

There are no gauges required in this theory and the QCD SU(3) is such a gauge. We have found that hadrons are these half integer spherical harmonic lobes.

The fractalness gave us the left handed Dirac doublet with electron mass and extremely small neutrino mass (section 16.2 above). Its new S matrix gave the W and Z particles masses as resonances. We have the muon and Tauon as 1S and 2S states respectively. We had the Lorentz covariance in our postulate of a geometrical point, eq.1.1: From the atomic physics analogy (and for example the Oxygen 3 lobes with 2 occupied by Hs in water) that 3 lobed $\psi^*\psi$ hybridization still exists even for $2P_{1/2}$. Note also that the 3 lobed character (hence the 1/3, 2/3 e charges) does not exist past the highest 2P state for at least some states (e.g., 3D states), thus past the top at ~140 GeV. These single sp^2 electrons do not annihilate with the paired positrons because they are in different states and the paired state is filled. The filled singlet positron electron (E&M) vacuum state and the free electron do not annihilate for the same reason. There are no gauges required in this theory and the QCD SU(3) is such a gauge. We have found that hadrons are these half integer spherical harmonic lobes. Thus we have all the ingredients, as a metatheory at least, to **construct the Weinberg Salam electroweak theory** for leptons**, but this time from first principles, our new Dirac equation pde.**

SU(3) Toy Model .
Here we derive the SU(3) toy model from our $2P_{3/2}$ and $2P_{1/2}$ (at $r=r_H$) analysis.
In Ch.19 we note that 1 in the new pde energy term (eg., in eq.19.7, $m_p=1$) is the mass of the 3 body proton $2P_{3/2}$ solution at $r=r_H$ for the baryons, the $\sqrt{(1+r_H/r)}$ (proton) ψ component (eg.,case1,case2 ch.19). This comes out of a rotation to the branch cut at 90° which means that r goes to r_H there. In the 4 body case we replace the +e and -e ψ components in the Dirac equation four component eigenfunction with those of the $\sqrt{(1-r_H/r)}$ (neutron). In that case the $2P_{1/2}$ implies an extra e or a -e in the case of the antineutron. or more generally also has a 0 e in the case of the proton, so 3 charge states. These values of q in the psi in the new pde then implies SU(3) The derivative in the new pde of e^{iqA} then gives SU(3) as a coefficient of the usual Dirac SU(2) eigenfunctions. In which case *we have derived the core of QCD which is* SU(3) in the exponent of the e^{iqA}. SU(3) gives the effects of this *added internal* e, so gives the **internal** eigenstates of baryons heavier than a proton.
Externally for matter all we see is a 0e neutron and +e proton which is a SU(2) 2 state in the new pde in which case *we have derived isospin* (Yang Mills).ie., |1/2,1/2> and |1/2,-1/2> where the second term in each bra is T_3. Note that SU(3) must be used to obtain the internal components before isospin can be used on the result.
Therefore SU(2) isospin can be implemented using the Clebsch Gordon coefficients.
For example:

$|I=1,I_3=1\rangle=pp$ (1)

$|l=1,I_3=0\rangle=\sqrt{.5})(pn+np)$ (2)

$|I=1,I_3=-1\rangle=nn$ (3)

$|I=0,I_3=0\rangle=\sqrt{(.5)}(pn-np)$ (4)

The deuteron has an isospin of I=0 the pion has isospin =1.
The reaction rate is then
$\sigma=|\text{amplitude}|^2=\text{sum over I } |\langle I',I_3'|A|I,I_3\rangle|^2$

Summary
We already have the left handed Dirac doublet from above and thus the U(1)XSU(2) theory can be constructed. More generally note the Dirac equation gammas contains the Pauli matrices that

obey quaternion algebra. The epsilon gives us a extra degree octonian of freedom whose tensor product can be used to generate E8.Lie group for rotational invariance of equation 1.2.

Composite System |1>|2>|3>

It is well known that (and also implied by the new pde) for the composite system of two electrons |1>|2> you get, from the analysis of the invariance of the resulting Casimir operator J^2, the resulting state $|J_A,J_B,J,M>$ with combined operator $J_A+J_B=J$. Using the resulting Clebsch Gordon coefficients we find the decomposition $2\otimes2=3\oplus1$, m=1,0,-1 ortho triplet state and singlet para state, which indeed are well known.(eg., Zeeman or Paschen Back line splitting). But for a third spin 1/2 particle we have |1>|2>|3> and so the Clebsch Gordon coefficients imply the decomposition $(2\otimes2)\otimes2=(3\otimes2)\oplus(1\otimes2)=4\oplus2\oplus2$ so that **three spin 1/2 particles** group together into **four spin 3/2** and only two spin 1/2.s, **6 states** altogether. Note then the majority 3/2 (trifolium core) states!

Recall also that the $2P_{3/2}$ solution to new pde at $r=r_H$ gives trifolium shape, fills first.

Also those ultrarelativistic plates contain such large B fields that the Paschen back effect at $r=r_H$ splits these states into the **6** udscbt states (flavors). The P lobes can't leave (asymptotic freedom) and P wave scattering implies the jets. Also the two positron plates only cross at the central electron position hence they don't repel and we thus are allowed a bound state even with two positrons. The positron and electron may rarely annihilate into a gamma ray but even if they do, because of the presence of the third heavy particle, that gamma ray will immediately create a electron and a positron replacement making this annihilation merely virtual.

Single Electron Probability:

A single electron in the trifolium implies that on average each of the 3 trifolium lobes has (1/3)e charge. (fractional charge).

Multiparticle Electron Probability:

For two positrons and one electron the probability of seeing a +(2/3)e lobe is twice that of seeing a -(1/3)e lobe ((2/3,2/3,-1/3 or uud)

For two positrons, an electron and an outlier electron the probability of seeing a -(1/3)e lobe is twice as high as a +(2/3)e lobe.(-1/2,-1/3,2/3 or ddu).the mass eigenvalues are gotten from the above Paschen Back mechanism associated with those plate fields.

Hence we have derived quark theory from a new pde that only contains one type of particle. -e,+e.

Thus given the derivation of the W, Z resonances (masses) from our new S matrix the linear transformations of A and B can be constructed for the Dirac doublet and resulting U(1)XSU(2). Given the tauon and muon (2S and 1S states respectively) we can proceed to derive the dynamical part of the electroweak Lagrangian density for the electron and electron neutrino and the lepton coupling to the W and Z. We could develop the conservation of charge and lepton number, etc… all the key achievements of electroweak theory. Thus we can essentially reverse engineer, derive, the Weinberg Salam Electro Weak interaction from the bottom up since we don’t have to postulate all these ingredients anymore (derived them from equation 1.9). Also since the $2P_{3/2}$ solution to that new Dirac equation implies quark SU(3) theory then we have the complete SU(3)XSU(2)XU(1) left Standard model theory. Note the W and Z masses and a spin0 charge 0 particle (of section 16.1) were resonances in the new pde S matrix and so do not require the Higgs boson. Thus we predict here that there is no Higgs particle and so the *Higgs will not be discovered*. Also postulate 1.1 implies a spin ½ new pde with then spin 0 is no longer as fundamental as spin ½. Also one motivation for the existence of the SUSY higher energy partners is the requirement for renormalization which does not exist here. With these two fundamental motivations for SUSY gone *we predict then that the SUSY higher energy partners will not be found.*

Reference

Arfken, *Mathematical Methods of Physics*, 3rd ed. Page 454
Enge, Harold, *Introduction To Nuclear Physics*, 1966, Addison Wesley, page. 45

C Meson Mass Derivation From Potential Of Chapter 14 And The New Pde eq.1.9
C Spherically Symmetric Wave Function Required

```
      PROGRAMFracsN
      DOUBLE PRECISION A,B,C,D,E,F,H,I,I1,J,KK
      DOUBLE PRECISION K1,K2,K3,K4,N1,N2,N3,N4,R,W,X,Y,Z
      DOUBLE PRECISION Y1,E1,E2,MM1,MM2,MM3,EE,JJ
      integer N,M,M1
      DIMENSION EE(400)
C     Variational principle on E with respect to I and Y1,
C     RungeKutte on D equation 18.1. Y=2 width Deuteron
C     pion oscillation resonance modeled between 0 and Y=2.
      H=0.001
      mH=2 !harmonic number for oscillation inside Y=2.
C     mN=1 gives pion 0 and K+-,mN=2 gives pi+- and Ko resonance
      ep=0.08*mH   !pion 1st and 2nd harmonic resonance added to Y1
      W=1.0+ep !pion mass added to nucleon.
      J=0.0  !spin 0 mesons
      X=0.0001   !mass energy increments
      I1=100000000.0
      A=0.0
      B=0.0
      C=0.0
      E=0.0
      KK=78.8   !gives MeV energy units
      JJ=J*1.
      Y1=2.0+ep  !pion  increases Y1.
50    D=.0000001
      I1=0.0
      F=.0000001
      Y=Y1
60    R=Y
       V=1.0/(1.0+ep-R) !chapter 14 potential for spin 0
       E1=E
       K1=((W-E-V)*F)+(((J-0.5)/R)*D)
       N1=((E+W+V)*D)-(((J+1.5)/R)*F)
       R=R+(0.5*H)
       V=1.0/(1.0+ep-R)
       K2=((W-E-V)*(F+(0.5*H*N1)))+(((J-0.5)/R)*(D+(0.5*H*K1)))
```

```
      N2=((E+W+V)*(D+(0.5*H*K1)))-(((J+1.5)/R)*(F+(0.5*H*N1)))
      K3=((W-E-V)*(F+(0.5*H*N2)))+(((J-0.5)/R)*(F+(0.5*H*K2)))
      N3=((E+W+V)*(D+(0.5*H*K2)))-(((J+1.5)/R)*(F+(0.5*H*N2)))
      R=R+(.5*H)
      V=1.0/(1.0+ep-R)
      K4=((W-E-V)*(F+(H*N3)))+(((J-0.5)/R)*(D+(H*K3)))
      N4=((E+W+V)*(D+(H*K3)))-(((J+1.5)/R)*(F+(H*N3)))
      E=E1
      F=F+((H/6.0)*(N1+(2.0*N2)+(2.0*N3)+N4))
      D=D+((H/6.0)*(K1+(2.0*K2)+(2.0*K3)+K4))
       I=(F*F)+(D*D)
100   I1=I1+(I*(R+(0.5*H))*(R+(0.5*H)))
      IF((abs(R-1.0-ep)).LT.(0.9*H))THEN
      Y=Y-(2.0*H)
      GOTO 60
      ENDIF
     Y=Y-H
     IF(Y.LT.0.0)THEN
      GOTO 200
      ENDIF
      GOTO 60
200   E=E+X
      C=I1
      IF(B.LT.A)THEN
      GOTO 310
      ENDIF
      GOTO 312
310   IF(C.GT.B)THEN
      ENDIF
312   IF(B.GT.A)THEN
      GOTO 315
      ENDIF
      GOTO 320
315   IF(C.LT.B)THEN
      print *,'  '
      print *,'E=',(E-X)*KK,' J=',J,' max I'
      ENDIF
320   IF(E.GT.8.0)THEN
      GOTO 349
      ENDIF
      A=B
      B=C
330   GOTO 50
349   print*,'program finished'
350   stop
      end
```

C **Results for** spin 0,L=0 are
C For mN=1 get 135MeV π^o and 493$K^\pm$ for resonance with 1 meson.
C For mN=2 get 139Mev $\pi^\pm$ and 497Mev K^o for resonance with two mesons in ordinary nuclear matter nucleus would split before K energy created. In a neutron star however K s could be created.

This fortran computer program only requires a few seconds to run on a PC. On the other hand lattice gauge theory programs (assuming a SU(3) lattice) require massive computing power and really do not duplicate high energy liquid state strong interactions anyway.