

# Part IV

## 4D *IS* Quantum Mechanics

### Chapter 21

#### 21.1 QM is The Result of 4 Dimensionality

Review: Copenhagen interpretation is merely 4D

Recall in Chapter 1 we postulated a 2D geometric point  $dZ=dr+idt$ . We add 2D observer to 2D object to get 4Degrees of freedom and the new (Dirac) pde  $(\sum_{\mu} \sqrt{\kappa_{\mu\mu}} \gamma^{\mu} \partial\psi/\partial x^{\mu} - \omega\psi=0)$  with its  $\psi$ .

Thus in adding the 2D observor you have collapsed the wave function to the observed  $\psi$ : hence the Copenhagen interpretation(CI) of QM is essentially a 4D theory. Also this postulate does not allow hidden variables because that point postulate 1.1 is fundamental, can't get any deeper into some hidden variable realm lets say. Thus the introduction of the observer results in 4D and the collapse of the wavefunction to  $\psi$ . Thus the

**Copenhagen interpretation of quantum mechanics is really a statement of 4D.**

By the way this is all well and good in free space since the  $\kappa_{\mu\nu}$  are trivial then. But in general the eq.1.5, 1.6 constraints on the  $\kappa_{\mu\nu}$  s make this still an intrinsically 2Degree of freedom theory.

#### 21.2 Contrast with Other QM Interpretations

We begin by defining:

U=time development operator= $e^{-iHt/\hbar}$  .

$\uparrow$ =spin up

$O_i$ =initial observer state

$O_{uf}$  =state of observer for spin up after observation

For the moment we return to the nonrelativistic approximation to the Dirac equation, the Schrodinger equation representation. Thus in that representation, if an atom is originally in a superposition of  $\alpha(\uparrow)+\beta(\downarrow)$ , then the state resulting from the observer interaction is:

$$U((\alpha(\uparrow)+\beta(\downarrow))\otimes|O_i\rangle) = \alpha|\uparrow\rangle\otimes|O_{uf}\rangle + \beta|\downarrow\rangle\otimes|O_{df}\rangle$$

If  $\alpha|\uparrow\rangle\otimes|O_{uf}\rangle$  is what is then measured then in the (dominant) Copenhagen interpretation  $\beta|\downarrow\rangle\otimes|O_{df}\rangle$  is just (arbitrarily) dropped and we say that the wave function has collapsed to the final measured value  $\alpha|\uparrow\rangle\otimes|O_{uf}\rangle$ . Unfortunately in this interpretation this postulated wavefunction collapse by definition happens instantaneously over all space so implies nonlocality and superluminal communication. In many worlds (MWI) on the other hand we don't drop the second term and simply ignore it (call it the second world of many such possible worlds if you want, (Tegmark 1997) even though the second state adds energy to the system (these are not virtual states) and so *leads to a violation of the conservation of energy*. But we still do avoid those wave function collapse conundrums with MWI, but at what a cost. In Bohm's interpretation we add adhoc hidden variables to the system along with other adhoc complications(recall we don't allow for hidden variables here anyway). Thus all three QM interpretations appear either adhoc or incorrect. There is a 4<sup>th</sup> way with no such liabilities that is discussed extensively in the first part of this book. Just note that we can construct the Copenhagen interpretation by bringing in the 2D observer and therefore the collapse to  $\psi$ . The

Copenhagen interpretation just becomes 4D, a simple, elegant understanding of what otherwise, over the past 70 years at least, has been taken to be a pretty mysterious concept. Furthermore the Schrodinger equation is nearly parabolic differential equation and so boundary conditions appear to apply (nearly) instantaneously whereas the Dirac equation is near hyperbolic and the path integral gives light speed limited information propagation. Since the Schrodinger equation is a special low energy case of the Dirac equation then velocity of information transmission is still limited to below  $c$  propagation by the Dirac equation. Thus entangled states do not give information moving faster than  $c$ . This won't let us get away (anymore) with claiming that superluminal communication is a result of wavefunction collapse created by bringing in the observer to create the 4D. Also we needn't postulate wave function collapse since bringing in the observer gives  $2+2=4$  degrees of freedom which are then restrained by saying the Schrodinger boundary conditions are responsible and thereafter constraining them by the Dirac equation.

**Klein paradox:** does not occur with equation 1.9; that generalized Dirac equation. There are always reflection eigenfunction solutions at very high potential barriers that are smaller amplitude than the input eigenfunctions which is not true for the old Dirac equation.

### 21.3 Covariant Gaugeless Quantization of the Electromagnetic Field

The standard quantization of the electromagnetic field treats the vector potential very differently from the scalar potential so the standard method is not covariant. Also, that method requires the Coulomb gauge and gives a sum of plane wave solution at the end and thus there is no hint of the pulse nature, lets say of a gamma ray photon (click on a Geiger counter lets say). Our remedy for these many problems is to simply add a scalar potential term to the standard vector potential (Merzbacher QM, page 556, 2<sup>nd</sup> ed. Wiley) quantization term; i.e., sum to 4 *instead* of 3 in:

$$V^{(=)} = \sqrt{4\pi\hbar c^2} \frac{1}{L^{3/2}} \sum_{k=1}^4 \frac{1}{\sqrt{2\omega_k}} [a_1(k)e_k^{(1)} + a_2(k)e_k^{(2)}] e^{ikr}$$

Also given the Heisenberg equations of motion (chapter 10)

$$i\hbar \frac{da(k,t)}{dt} = [a(k,t), H] \quad (21.1)$$

4 vector quantization of the electromagnetic field

$$V^{(=)} = \sqrt{4\pi\hbar c^2} \frac{1}{L^{3/2}} \sum_{k=1}^4 \frac{1}{\sqrt{2\omega_k}} [a_1(k)e_k^{(1)} + a_2(k)e_k^{(2)}] e^{ikr}$$

Then goes to the time dependent form

$$V^{(=)} = \sqrt{4\pi\hbar c^2} \frac{1}{L^{3/2}} \sum_{k=1}^4 \frac{1}{\sqrt{2\omega_k}} [a_1(k)e_k^{(1)} + a_2(k)e_k^{(2)}] e^{ikr-i\omega t} \quad (21.2)$$

Where the usual 3 vector potential components are given by

$$A^{(=)} = \sqrt{4\pi\hbar c^2} \frac{1}{L^{3/2}} \sum_{k=1}^3 \frac{1}{\sqrt{2\omega_k}} [a_1(k)e_k^{(1)} + a_2(k)e_k^{(2)}] e^{ikr-i\omega t}$$

And the scalar component is given by

$$V^{(=)} = \sqrt{4\pi\hbar c^2} \frac{1}{\sqrt{2\omega_k}} \frac{1}{L^{3/2}} [a_1(k)e_4^{(1)} + a_2(k)e_4^{(2)}] e^{ikr-i\omega t}$$

Where the  $e_4^1$  and  $e_4^2$  are defined so that, along with the usual transverse polarization vectors (to the  $k$  vector), the orthogonality of the 4 vectors  $e_k$  and  $k$  results in

$$\nabla \cdot A(r, t) - \partial V / \partial t = 0 \quad (21.3)$$

The  $e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}$  term implies a wave equation given by

$$\left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) V = 0, \left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) A = 0 \quad (21.4)$$

Here  $B = \nabla \times A$ ,  $E = -\nabla V$

Note that 21.3 and 21.4 is the Lorentz gauge explicitly covariant form of the Maxwell equations (section 17.3). But 17.3 are the actual physical (Maxwell) equations in potential form and so are not gauged here (section 12.3). Thus, this is a gaugeless and covariant quantization of the electromagnetic field.

### 21.4 Beat Frequency Envelope and the Uncertainty Principle

Equation 21.2 gives plane wave sums of energy  $\hbar\omega$  with no hint, for example, that the sum is the pulse of a gamma ray click on a Geiger counter. It could be anything. So, how then can plane waves ever give rise to local clumps of energy (called ‘photons’ in the in the Maxwell equation case)? In that regard, recall that the probability density interpretation of  $\psi^*\psi$  motivates the uncertainty principle. But the uncertainty principle in time and hence in frequency) for multiple emitters allows waves (here the plane wave solutions to Maxwell equations) of differing frequency  $\Delta\omega = \frac{1}{2}(\omega - \omega')$  in a plane wave to form “beats” of the plane wave. Hence the origin of photons, here as merely these beats!!!! The prosthaphaeresis formula

$\cos \omega't + \cos \omega t = 2\cos(1/2(\omega' + \omega) t)\cos(1/2(\omega - \omega')t)$  then gives this beat frequency as  $\Delta\omega$ .

Next we show that the energy of one of these uncertainty principle beat frequency envelopes is  $E = \hbar\omega$  for the SHM and for the resulting E fields this oscillation creates (as in an antenna). Note these are SHM  $\psi$ s so we can then use the SHM Schrodinger equation Hamiltonian expectation values. The E&M electric fields also must have the same amplitude dependence and the consequent energy transitions must also be the same as the SHM creating them thus constraining integration limits and amplitudes. To find the energy of a single beat frequency envelope given by this use of the uncertainty principle we can then substitute the  $\psi$  into the Schrodinger equation expectation value of the Hamiltonian  $E = \int \psi^* (-i\hbar d/dt) \psi d\tau$ . We correct for the cosine not being the shape of a minimal uncertainty wave packet (with the .2067 integration limit below) and take the integrations symmetrically about the center so the  $\cos \omega t$  (for average  $\omega$ ) comes out of the integral. We normalize a one dimensional  $\psi$  over beat length  $L$  which is the minimal  $\Delta x$  uncertainty corresponding to the above  $\Delta\omega$  ( $n=2$  for  $\Delta x = 2L$ ).

$$\frac{\int_0^{G\Delta\omega\Delta t / 2(\max)} f(\omega) \cos(\Delta\omega\Delta t / 2) d(\Delta\omega\Delta t / 2)}{\Delta\omega\Delta t / 2[\max]} = \frac{\int_0^{.2067} f(\omega) \cos \theta d\theta}{.25} = 1.0$$

by setting the integration limit equal to .2067 and  $f(\omega) = 1$

$$E = \int \psi^* (-i\hbar d/dt) \psi d\tau = \int [\cos(\omega t)] \cos(\Delta\omega\Delta t) / G\sqrt{\Delta x}^* (-i\hbar d/dt) [\cos(\omega t)] \cos(\Delta\omega\Delta t) / G\sqrt{\Delta x} d\tau =$$

$$\begin{aligned}
& -i \int_{-\Delta x/2}^{\Delta x/2} \left[ \frac{\cos(\varpi t)}{\sqrt{\Delta x}} \left( \frac{\int_0^{G\Delta\omega\Delta t/2(\max)} \cos(\Delta\omega\Delta t/2) d(\Delta\omega\Delta t/2)}{\Delta\omega\Delta t/2[\max]} \right) \frac{d}{dt} \left( \frac{\cos(\varpi t)}{\sqrt{\Delta x}} \left( \frac{\int_0^{G\Delta\omega\Delta t/2(\max)} \cos(\Delta\omega\Delta t/2) d(\Delta\omega\Delta t/2)}{\Delta\omega\Delta t/2[\max]} \right) \right) \right] dx \\
& -i \left( \int_{-\Delta x/2}^{\Delta x/2} \operatorname{real} \left( \frac{e^{-i\varpi t}}{\sqrt{\Delta x}} \right) \frac{d}{dt} \left( \operatorname{real} \frac{e^{i\varpi t}}{\sqrt{\Delta x}} \right) dx \right) i =
\end{aligned}$$

Taking in general imaginary sine and real cosine components of the Schrodinger equation plane wave and multiplying both sides by  $\hbar$  to set the energy units:

$$-i \hbar \left( \int_{-\Delta x/2}^{\Delta x/2} \left( \frac{e^{-i\varpi t}}{\sqrt{\Delta x}} \right) \frac{d}{dt} \left( \frac{e^{i\varpi t}}{\sqrt{\Delta x}} \right) dx \right) i = -i \left( \varpi \int_{-\Delta x/2}^{\Delta x/2} dx / \Delta x \right) i = E = \hbar\varpi$$

which is the energy of the photon. You might even say that the uncertainty principle (when applied to a plane wave) then demands the existence of photons and the other particles.

Thus, in contrast to the usual 1) lack of covariant quantization, 2) Coulomb gauge E&M field quantization, 3) lack of specificity on the pulse nature of the photon, 4) many and varied interpretations of QM, etc.,

- 1) Here the Quantization of the E&M field is Covariant
- 2) The Quantization of the E&M field doesn't require a gauge
- 3) Here we have the pulse nature of the photon as well (e.g., gamma ray click on a Geiger counter)
- 4) The simplest way to understand (Copenhagen interpretation) quantum mechanics is that it is 4D, the mystery is solved.

## 21.5 More Consequences of Reducing the Dimensionality: Possible Source of the Uncertainty Principle (and *still* ONE postulate)

That standard proof of the uncertainty principle is 1) Bohr postulate of  $\psi^*\psi$  as the probability density and then 2) using the Cauchy Schwarz inequality on the integral of the expectation value of  $(x-\Delta x)(p-\Delta p)$ . This has the appearance of proving what you are assuming since the Bohr postulate implicitly contains this uncertainty idea. Perhaps there is a deeper meaning to the uncertainty principle, one in which that integral provides a uncertainty  $\Delta x$  region analogous to the Rayleigh region in optics, hinted at by the fact that both are wavelength dependent effects. In that regard why can't truth (i.e., psf position and intensity information) be extracted from deep within the Rayleigh region even for high SNR?

The answer to the above question is that we must "reduce the dimensionality" of the problem to extract this information by finding ridges (in intensity space) using simple slope thresholds along circles around the intensity maximum. These ridges then form lines, which are *one* dimension instead of the 2D image, *hence the reduction in dimension*. Thereafter we initialize our least squares search on these lines. This is the solution to the problem of super-resolution. So we never test on the ghost images and hit right up against the Cramer Rao lower bound. I did my own experiments with real optical systems to verify this method worked (2007 mdsea paper the method is called DRCSO).

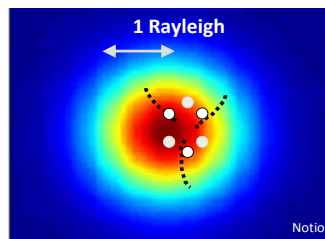
In any case within the diffraction limit those "ghost" images, or artifacts, seem to be connected to the truth configurations via simple SO(2) or translational group symmetries. The simpler the group symmetry here the smaller the least squares differences suggesting a group theoretical configurational origin of that subrayleigh image ambiguity problem.

The reason for mentioning this configuration ambiguity problem here is that it may then be the underlying source of the Bohr-Born postulated probability density  $\psi^*\psi$  (so implied uncertainty) which here then is seen to originate in that CSO position ambiguity (remedied by DRCSO) so it then is a derived result, not postulated. Therefore *we still have only ONE postulate* eq. 1.1, Chapter 1.

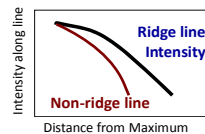
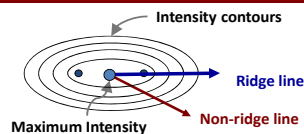
### The Dimensional Reduction Set

**Maker's observation:** Source centers *tend* to lie along ridge lines that have negative curvature in all directions— Saddle lines excluded.

- The DR Set is a sampling of points along ridge lines
- Initialize optimization by selecting points from the DR Set
- There must be enough Signal-to-Noise to find the ridge lines

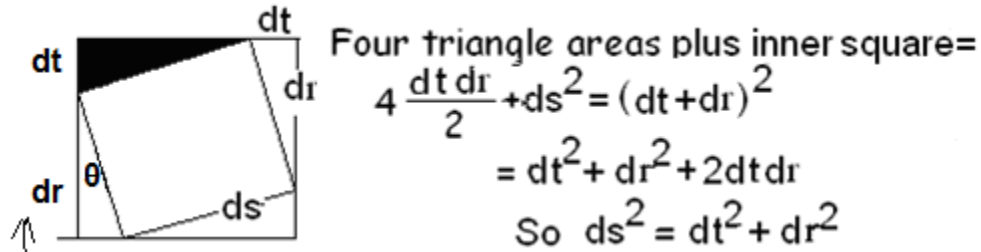


**Ridge lines of interest are lines of slowest monotonic descent**



**21.5**  $dr+dt=\text{constant}$  and  $\delta(ds)=0$  Implies  $|dr|=|dt|$

Recall section 1.1.



Here  $dr+dt=\text{constant}$ . Within this constraint  $ds$  is smallest, thus an extrema when also  $dt=dr$ ,  $\theta = 45^\circ$  as in equation 1.2a.. Also note that  $dr+dt=\text{constant} = ds/\sqrt{2}$  at  $\theta = 45^\circ$ . The hypotenuse defines the invariant  $ds$  in equation 1.3.

**Figure 21-1**

Note when  $dr+dt= \text{constant}$  and  $\delta(ds)=0$   $\text{constant}=ds\sqrt{2}$  and  $dt=dr$ .

**Summary of Book**

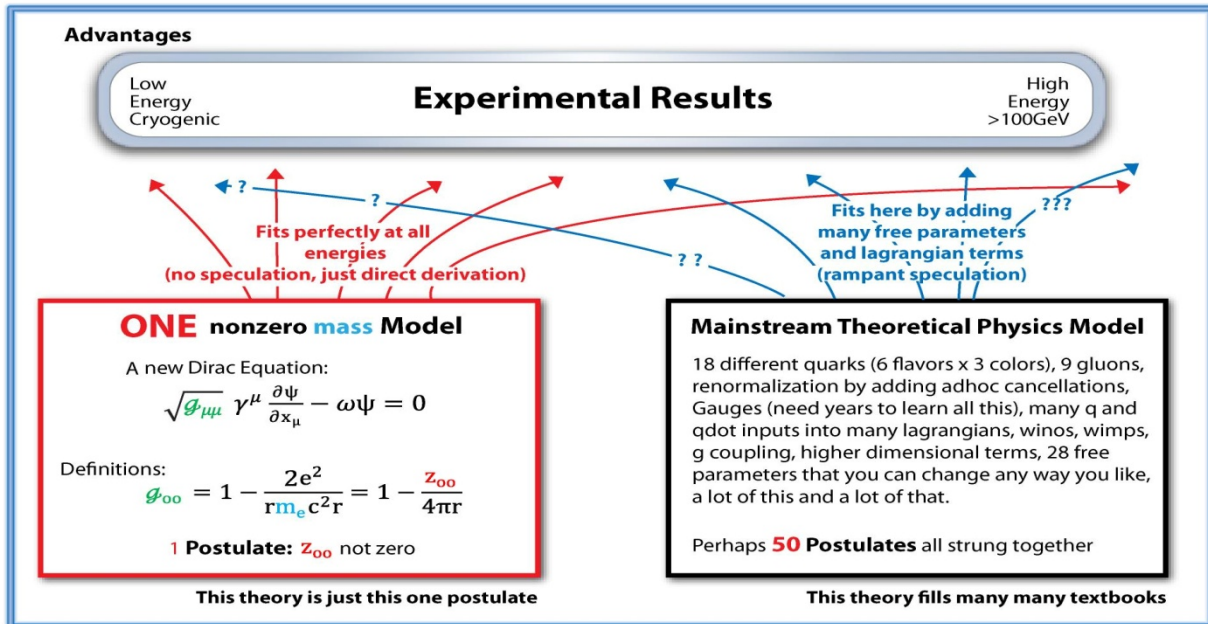
Postulate geometrical point:  $dZ=dr+ idt$

implying a new pde:

$(\sum_{\mu} \sqrt{\kappa_{\mu\mu}} \gamma^{\mu} \partial \psi / \partial x^{\mu} - \omega \psi = 0)$  with  $\sqrt{\kappa_{00}} = \sqrt{(1-2e^2/rm_e c^2)} \equiv \sqrt{(1-r_H/r)} = 1/\kappa_r$  ..

Solve this new pde in domains  $r > r_H$ ,  $r \approx r_H$ ,  $r < r_H$

There can be no more rigorous or fundamental ways of doing theoretical physics!



**Figure 21-2**

**References**

Bell J.S., *Speakable and Unspeakable in Quantum Mechanics*, 1987, Cambridge, pp.84  
Max Tegmark 1997, arXiv:quant-ph/97/09032v1 15Sept 1997